The Formation of Social Groups Under Status Concern

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Abstract

I study the interaction of two forces in the formation of social groups: the preference for high quality peers and the desire for status among one's peers. I examine their equilibrium effects under different market structures and find that status concern reduces the potential for and benefit of sorting - both for a social planner and a monopolist - but the interaction of preference for quality and status can make the exclusion of some agents a second-best outcome. Even in settings with complementarities, price discrimination and screening can be necessary to facilitate sorting and increase welfare. Nevertheless, positional concerns can be beneficial for welfare if they provide a sufficient incentive to engage more with one's group and thus increase positive spillovers. In those cases, welfare is higher if individuals have at least some degree of status concern, even if the welfare measure ignores such relative comparisons.

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1 Introduction

When people interact in a social environment, whether it is at work or school, in clubs or in their neighbourhood, social spillovers tend to play an important role. At work, cooperation with colleagues might be essential, at school and university, studying with peers can promote understanding and enhance the learning experience. In many of these situations, we would like to be surrounded by 'strong' peers as their ability influences the benefit we gain from the interaction. At the same time, we might want to be someone with a relatively high standing in the group. This presents a clear tension: the stronger the peers, the lower one's own standing. Consider moving house and choosing a new neighbourhood: the decision where to live is, among other factors, most likely influenced by the quality of public services, the valuation for these, and the cost of living in the different areas. But in addition, one might be worried about the relative status among the potential neighbours. A better public library may not compensate for the discomfort caused by being one of the lowest earners. This paper develops a model to explore the importance of these peer effects in the formation of social groups, very much in the spirit of Frank (1985). It addresses the questions what groups can be formed and what groups might be offered by a social planner, monopolist, or competitive firm when individuals care about both the quality of peers, as well as their standing within their group. The focus lies on two key aspects: social sorting and exclusion. It is explored how status concern affects the segregation of individuals (how they can be sorted into groups). And it is examined what status concern implies about social exclusion, addressing the question how many individuals might not be offered any social group.

In the model, a large number of agents observe a set of prices for group membership and simultaneously decide which group to join and how much to engage with the group. Agents are heterogeneous in their type: a one-dimensional variable, for example, income. The agents' payoff is determined by the composition of the group, the extent to which they socialise within the group, the membership price, and their own type. In particular, two aspects of a group determine the social spillovers: it's *quality* - a function of the other members' types and engagement choices - and the *status* of an individual - the rank in the distribution of types in that group.[1] It is assumed that there is a positive interaction between type and the characteristics of a group. Agents with higher type value quality and rank more; just like high earners might care more about the quality of schools as well as their own social status.

[1]While social status can have multiple dimension, Heffetz and Frank (2011) argue that it is inherently positional; a form of 'rank'. The simplification here is that people agree on the ranking. There is evidence that this is often the case. See Weiss and Fershtman (1998) for a survey of the relevant literature.

The analysis also specifically explores the effects of status concern under different market structures. In these cases, the agents' choice set is determined by a seller or 'provider' that offers a set of groups. This provider could, for example, be a local authority deciding on the number and types of schools in the district and their tuition fees; or a firm developing a housing project, choosing how inclusive the development should be. The provider might act as a benevolent social planner - maximising welfare - or as a monopolist - maximising profits from membership fees. This is further contrasted against outcomes in a competitive market.

It is shown that status concern reduces the potential for, as well as the benefit of segregation, i.e., sorting the population into into several, separate groups. Whether a provider maximises welfare or revenue, status concern leaves less room for manoeuvre. It is a force for heterogeneity within groups as it limits how many distinct groups can be offered in equilibrium. There might be no prices that make a given group structure incentive compatible, even though such prices exist if agents only care about peer quality. Sorting cannot be arbitrarily fine, even if it is costless. Equilibrium groups take the form of non-overlapping intervals and, as is shown, the number of such intervals must be finite. If status concern is sufficiently strong, no sorting can be achieved as only a single group can be offered in equilibrium. If welfare does not take into account positional concerns, then those restrictions cause a welfare loss that is increasing in the magnitude of status concern.

Beyond posing restrictions, status concern also renders sorting less beneficial - both in terms of revenue and welfare (if it does enter the welfare objective). If status concern is sufficiently strong, offering a single group is welfare- and profit maximising, despite the complementarity between individual type and group quality. While seemingly aligned, restrictions on group provisions and changes in the optimal group structure do not generally match, leading to inefficient equilibrium outcomes for intermediate levels of status concern. In contrast, if individuals only care about the quality of a group, any interval partition can be achieved in equilibrium and arbitrarily fine sorting might be optimal.[2]

As a second key observation, the interaction between quality and status concern can make the exclusion of some individuals from any social group (by setting membership prices sufficiently high) a second-best outcome. This is true even though all individuals benefit in principle from (any) group membership. If agents and social planner care only about quality or only about status, this cannot be the case; at least not in a large enough population. In this sense, the concern for status *and* quality can

[2] Board (2009) finds in a closely related setting without status concern that for sufficiently convex quality functions, full separation is a welfare and profit maximising equilibrium.

lead to social exclusion. In the context of the education example, even an authority maximising utilitarian welfare might set tuition fees such that some students choose not to acquire higher education. Maybe surprisingly, it is shown that in some cases a monopolist charges a lower price for the lowest-quality group, thus excluding fewer agents than a social planner, benefiting lower types.

These two observations imply that even with complementarities, price discrimination and screening can be necessary to facilitate sorting, prevent social exclusion, and increase welfare. The paper explores to which extent these can counteract the inefficiencies caused by status concern. In contrast to the conclusions drawn in Schelling (1971), Arrow (1998), and Ellickson et al. (1999), with status concern price discrimination can remedy inefficient integration, rather than segregation. Furthermore, incentive compatibility can require redistribution across groups.

Despite these negative effects, it is shown that some degree of status concern can nevertheless yield positive welfare consequences. If status provides an incentive for a large enough fraction of a group to engage more, it can raise group quality and hence positive spillovers. Reminiscent of the literature on contests and organisational design (Moldovanu and Sela, 2001; Moldovanu et al., 2007), positional concerns can lead to more efficient outcomes despite the restrictions on sorting - even if the welfare objective attaches no weight to status concern.

Regardless of the abstract nature of the model, several implications in relation to the literature on social groups might warrant further consideration. When agents care about their relative rank, we should expect groups to be less segregated and more heterogeneous in their membership base. In the empirical literature on Tiebout sorting - the sorting of individuals into different communities based on their preferences for public goods (Tiebout, 1956) - it is often noted that communities are much more similar across and diverse within than should be expected (Persky, 1990; Epple et al., 2001; Calabrese et al., 2006). This aligns with the finding on sorting here. Status concern can, for a similar reason, have important implications when identifying peer effects. If we try to measure the magnitude of complementarities by the degree of sorting, we need to consider how important status considerations are. An absence of positive sorting can indicate strong rank preferences rather than the absence of complementarities. Status concern can also offer an additional perspective on the effects of redistribution. Subsidies, such as housing vouchers, may among many other effects (Chetty et al., 2016; Davis et al., 2021), help to facilitate more efficient sorting. But such transfers may thereby reinforce rather than reduce segregation and inequalities.

1.1 Related Literature

The model draws from three closely related theoretical papers: Board (2009) investigates the optimal monopoly pricing of social groups when the agents' types determine the quality of the group. His analysis shows that, independent of the exact nature of the quality function, the monopoly provision is too segregated and excludes too many agents. Similarly, in Rayo (2013) a monopolist sells 'group memberships' in the form of status categories that allow agents to signal their types to each other. This can lead to pooling for some subsets of agents and full separation for others. Both models can be interpreted as agents having preferences over local quality (the quality of their social group) and/or the global status in the population that results from it. In the model here, agents have preferences over local quality and local status, which is jointly determined by their choice of group membership. Levy and Razin (2015) investigate the link between inequality, sorting, and preferences for redistribution. In an environment where agents have preferences over the mean type in their social group, they explore how inequality affects benefits from sorting. They show that in relatively equal societies, full redistribution is preferred to sorting by a large majority. This paper demonstrates how status concern can significantly alter some of these conclusions.

Taking a broader perspective, the analysis relates to two main themes: positional concerns and the provision of (local)-public goods. Following early ideas of conspicuous consumption (Veblen, 1899), the literature has subsequently explored the impact of relative comparisons on consumption and savings decisions (Duesenberry, 1949; Cole et al., 1992; Corneo and Jeanne, 1998; Becker et al., 2005), the welfare implications of different income distributions (Hopkins and Kornienko, 2004), urban sorting (Ghiglino and Nocco, 2017), and strategic interactions more generally (Haagsma and van Mouche, 2010). Frank (1985) addresses the connection between status concern and sorting. And Ray and Robson (2012), as well as Robson (1992), focus particularly on status as the rank in a distribution of a one-dimensional characteristic; similar to how it is defined here. With a stronger focus on ordinal comparisons, the literature on contests has examined status as a way to incentivise performance. Moldovanu et al. (2007), for example, look at the optimal partition of agents into status categories. The model here shares the zero-sum nature of those allocations. The literature on networks delivers many additional insights by focusing on the specific structure of social interactions. Ghiglino and Goyal (2010), for instance, analyse conspicuous consumption in a networked exchange economy and find that relatively less well-off agents can lose from social integration. In a closely related model, Bramoullé and Ghiglino (2022) show how loss-aversion can lead to homogeneous levels of conspicuous consumption among heterogeneous agents. In a simplified but very tractable framework, Langtry

(2023) demonstrates how flat taxes can increase welfare, and how such a model can be useful in analysing trends in labour market inequality. As a key contribution to the literature on relative comparisons, the model here looks at the joint effects of sorting and social comparisons.

The importance of positional concerns is also validated empirically (Frank, 2005).[3] The role of relative income within a neighbourhood, for instance, has been investigated extensively in Luttmer (2005). Using data from the American Household Survey, the study finds that relative income changes have an effect of similar magnitude on life satisfaction as absolute ones, and - consistent with the model here - that the effect is stronger for people who socialise more with their neighbours. Bottan and Perez-Truglia (2022) highlight the importance of relative income in location choices of medical students. Most closely related to the trade-off investigated here, Clark et al. (2009) use data from a Danish household survey to demonstrate that respondents' economic satisfaction depends on neighbourhood income levels as well as local income rank .

There is a large body of literature on social spillovers and particularly the production and sharing of goods in clubs (Buchanan, 1965; Aumann and Dreze, 1974; Demange and Henriet, 1991) that blur the line between purely public and private goods; not unlike the quality of a group in the model proposed here.[4] In a general equilibrium setting, Scotchmer (2005) studies the pricing of clubs with heterogeneous agents. If groups can discriminate between relevant characteristics and thus effectively limit free movement of consumers, consumption externalities can be internalised, which resembles the welfare benefits from price discrimination demonstrated in this model.

The empirical literature on these spillovers is too rich to attempt even a cursory overview. I instead focus on one particular issue raised by Tiebout (1956): the endogenous sorting of agents in communities when preferences are heterogeneous. Tiebout has spawned a large literature that studies the provision of public goods by competing jurisdictions, which differentiate themselves through the public goods they offer and the taxes they charge. This differentiation should lead agents to sort efficiently (McGuire, 1974; Greenberg, 1983; Conley and Wooders, 2001). The empirical evidence, however, is mixed. The Tiebout model in its simple form predicts relatively homoge-

[3] Relative income significantly affects self-reported happiness (Alesina et al., 2004) and satisfaction at the workplace (Brown et al., 2008; Card et al., 2012). Positional concerns can also be a driver for migration decisions (Stark and Taylor, 1991), shape attitudes towards redistribution policies (Corneo and Grüner, 2002), and affect individuals' performance (Jemmott and Gonzalez, 1989). Furthermore, Ashraf et al. (2014) present evidence from an educational setting that people are aware of their relative standing, the salience of which influences choices.

[4] Social spillovers also received considerable attention in the context of networks, both in settings where efforts can be directed towards specific agents, (Bramoullé and Kranton, 2007; Bloch and Dutta, 2009; Baumann, 2021), and those where agents cannot discriminate between neighbours (Cabrales et al., 2011; Durieu et al., 2011).

neous communities (i.e., fine sorting). This prediction has been questioned (Pack and Pack, 1977), however, and communities appear to be more heterogeneous within and accordingly more similar across than predicted. This is discussed in Persky (1990) and more extensively in Epple et al. (2001) and Calabrese et al. (2006); the latter specifically shows how this disparity can be largely resolved when allowing for preferences over the composition of communities. The model here captures some of these aspects: depending on the group quality function, there can be an incentive to separate finely but the preferences over rank have an opposing influence, impeding such sorting.

2 Model

Group formation is modelled as a simultaneous move game in which a continuum of agents choose from a finite set of groups G. Each agent has a type w, drawn from a distribution F with support over an interval $[\underline{w}, \overline{w}]$. An agent can join (at most) one group, or not join any group at all ($\emptyset$), with the set of group related choices denoted by $A = G \cup \{\emptyset\}$. Furthermore, an agent can choose an *engagement level* $e \in \mathbb{R}_+$ that determines how active they are in the group and consequently how much they benefit from it. The action set is thus $A \times \mathbb{R}_+$. For each group $k \in G$, there is a membership price $p_k \geq 0$. The vector $\mathbf{p}$ contains all such prices. For a large part of the analysis, they are restricted to constants, meaning every member of a particular group pays the same (uniform) price. These could be the tuition fees at a university, or the membership dues of a social club. This is contrasted to a setting where some price discrimination is possible, with $\mathbf{p}$ a vector of functions of agents' characteristics. As an additional cost, engagement with a group is costly. Membership in a country club requires the right attire as well as the means to travel there. This is captured by the cost function $c(e)$.

Joining a group gives agents access to the peer effects generated by the other members. I distinguish between two types of spillovers: the *quality* of the group and the *status* of an agent. The quality q could be interpreted as a local public good, a form of social capital,[5] or a 'global' status good, like the prestige associated with membership in a particular group. It is determined by the members' characteristics and/or their engagement. Formally, q is a function from probability distributions over types and engagement levels to $\mathbb{R}_+$, with the assumption that for any given interval $[\underline{w}, \overline{w}]$, this function is bounded, i.e., $q \in [\underline{q}, \overline{q}]$. This can, for instance, be a statistic of the distribution of agents choosing the same group, like the average type, or their average engagement. In principle, it could also depend on the number (i.e., measure) of agents

[5]See, for instance, Coleman (1988) and Coleman (1990) for a characterization of 'social capital' and Sobel (2002) for a critical economic perspective.

choosing a group, to capture crowding effects. While this is not explicitly considered here, the set-up could be easily modified accordingly. As a second type of spillover, agents have preferences over their rank within their group. The status of an agent with type w in group k is $r_k(w) \equiv F_k(w)$; the CDF of types choosing k, evaluated at w. This closely follows existing definitions of status as, for example, in Ray and Robson (2012).

A *social group* itself is defined as an aggregate description of all individuals making the same choice in G and their engagement levels, i.e., a probability distribution with support over a subset of $[\underline{w}, \overline{w}] \times \mathbb{R}_+$ that admits a continuous CDF. For any particular social group $\mathscr{F}_k$, the smallest convex set containing its support over types is denoted by $[\underline{w}_k, \overline{w}_k]$, and we say $\underline{w}_k$ and $\overline{w}_k$ are the lowest and highest type in k. The corresponding marginal distribution over types is F_k. For a particular set of social groups to arise in equilibrium, they need to be consistent with the distribution of types in the population, and the choices they imply need to be individually optimal. To restrict attention to social groups that satisfy the required consistency, Definition 1 introduces the notion of *group structure*. Social groups implied by such a group structure will not only be consistent with F, but also non-overlapping in their support over types. The second property is not required but assumed for simplicity. It also emerges naturally with a suitable refinement.[6]

Definition 1. *[Group structure] A group structure is a triple $(I, \mathbf{e}, \mathbf{p})$, where $I = \{W_k\}_{i=1}^n$ is a partition of some subset $W \subseteq [\underline{w}, \overline{w}]$, $\mathbf{e} = (e_k)_{k=1}^n$ is a vector of functions with e_k mapping from $[\underline{w}_k, \overline{w}_k]$ to $\mathbb{R}_+$, and $\mathbf{p} = (p_k)_{k=1}^n$ is a vector.*

A group structure consists of a division of agents (I) into groups, their engagement choices ($\mathbf{e}$), as well as the membership prices of these groups ($\mathbf{p}$). It implicitly defines a set of social groups $\{\mathscr{F}_k\}_{k=1}^n$, where each marginal over types F_k corresponds to the appropriately scaled F, with the support restricted to a corresponding element of I, and the marginal over engagement determined by e_k. As not all types are necessarily in a group, the union of all sets in I can be a proper subset of $[\underline{w}, \overline{w}]$. To abstract from payoff irrelevant (measure 0) differences between partitions, I is restricted to contain Borel sets. This definition allows for an easy comparison of sorting properties through a comparison of I. $|I| = 1$, for instance, is referred to as an absence of sorting.

The preferences of agents are represented as follows:

$$U(w, e, k, \mathscr{F}_k) = \hat{U}(w, e, k, \mathscr{F}_k) - p_k = e \cdot [u(w, q_k) + v(w, r_k(w))] - c(e) - p_k.$$

Given e, utility is assumed to be separable in quality, status, and prices/ costs.[7] Sep-

[6]Social groups with overlapping support require indifference for a continuum of agents. With status concern, any perturbation in the distribution of types breaks this indifference.

[7]Maccheroni et al. (2012) present an axiomatic foundation for similar social preferences that are sepa-

arability in q and r is, however, not crucial for many results and mainly serves expositional clarity. In the absence of any restrictions, each individual optimally choose engagement. The following takes into account this maximisation:

$$U\big(w,k,\mathscr{F}_k\big) = \hat{U}\big(w,k,\mathscr{F}_k\big) - p_k = e^*_k(w)\cdot\big[u\big(w,q_k\big)+v\big(w,r_k(w)\big)\big] - c(e^*_k(w)) - p_k, \quad (1)$$

with $e^*_k(w)$ the optimal engagement level for type w in group $\mathscr{F}_k$.

As a convention, the choice of not joining any group is treated as a 'special' group $\mathscr{F}_\emptyset$, with the corresponding payoff equal to some $\underline{u} \in \mathbb{R}$. Since the benefits of group membership arise from the presence of others, $\underline{u}$ is also taken to be the payoff (excluding the membership price) from joining any 'empty' group, i.e., one chosen by a measure 0 set of players. Beyond this, the following assumptions apply:

Assumption 1 (Continuity, monotonicity, and complementarity)**.** *$u(w,q)+v(w,r)$ is continuous, at least twice differentiable, and increasing in w, q, and r. Furthermore, $u(w,q)$ is strictly increasing in w, $\frac{\partial^2}{\partial w \partial q}u(w,q) > 0$, and $\frac{\partial^2}{\partial w \partial r}v(w,r) \geq 0$.*

Assumption 2 (Single-crossing)**.** *Suppose $\mathscr{F}_l$ and $\mathscr{F}_h$ are social groups with $q_l \leq q_h$. If $u(\hat{w},q_h)+v(\hat{w},r')-u(\hat{w},q_l)-v(\hat{w},r) \geq \delta$ for some $\delta \geq 0$, $r',r \in [0,1]$, and $\hat{w} \in [\underline{w},\overline{w}]$, then this holds for all $w > \hat{w}$.*

Assumption 3 (Stand-alone payoff)**.** *The payoff $\underline{u}$ from the choice $\emptyset$ is such that $\underline{u} \leq \hat{U}(w,k,\mathscr{F}_k)$, for all $w \in [\underline{w},\overline{w}]$ and $\mathscr{F}_k$.*

Assumption 4 (Monotonic quality)**.** *Suppose $\mathscr{F}_l$ and $\mathscr{F}_h$ are social groups with $\overline{w}_l \leq \underline{w}_h$ and $\mathscr{F}_l(w,e) \geq \mathscr{F}_h(w,e)$ for all $w \in [\underline{w},\overline{w}]$, $e \in \mathbb{R}_+$, i.e., almost all members of $\mathscr{F}_l$ have lower type than those of $\mathscr{F}_h$ and engage less. Then $q_l \leq q_h$.*

Assumption 5 (Quadratic engagement cost)**.** *The engagement cost function takes the form $c(e) = ae^2$, with $a > 0$.*

The assumptions on monotonicity and complementarity capture that agents not only value quality and rank but that this valuation is increasing in their own type. This could be because people with higher socioeconomic status are better connected, and thus benefit more from social interactions (Campbell et al., 1986). They might also have a higher valuation for school quality and other public goods exclusive to their neighbourhood.[8] If status is instrumental in obtaining non-market goods, for instance

rable in a private and a positional component.

[8]Bayer et al. (2007) provide evidence that more highly educated households value the education characteristics of their neighbours more. And Barrow (2002) finds that the valuation for school quality is positively related with income and education. Furthermore, Mujcic and Frijters (2013) offer some more direct evidence that wealthier individuals have a stronger preference for rank.

in social contests (Cole et al., 1992; Corneo and Jeanne, 1998; Hopkins and Kornienko, 2004), then complementarity between type and status is equivalent to a complementarity between type and the non-market good.[9] The fact that these preferences tend to be observable through location choice implies that they are not entirely obscured by status concerns. This motivates Assumption 2, which states that if an agent of type w prefers higher quality over a given rank trade-off, then this must also be the case for all higher type. Otherwise, status concern might impede sorting simply by assumption. Assumption 3 makes joining any group beneficial - at least at $p = 0$. And Assumption 4 ensures that the function q captures the idea of peer quality. If all players in a sports team have higher ability than even the best player in another and put at least as much effort into playing, the team itself should be better. Finally, Assumption 5 ensures a unique interior solution for $e^*(w)$ and extends the single-crossing assumption to $\hat{U}$ (Lemma 3, Appendix A.1). Most results do not specifically rely on the quadratic form and apply to other cost functions that ensure these two properties.

We can now introduce the equilibrium notion for the (sub-)game described so far, in which the agents observe prices, decide on a group, and choose their engagement level. The analysis focuses on pure-strategy Nash equilibria. Following Definition 1, a group structure $(I, \mathbf{e}, \mathbf{p})$ implies an assignment of agents to groups. To make this explicit, let $g(w)$ denote a function that maps from $[\underline{w}, \overline{w}]$ to A, with the following properties: g assigns the same choice in G to all types in a particular element of I, a different group choices across elements, and no group ($\emptyset$) to all types outside of I.[10] I refer to this as an assignment function corresponding to I.[11] This generates a set of social groups. If the group assignments and engagement levels prescribed by g and $\mathbf{e}$ are incentive compatible given those social groups, this is called an equilibrium:

Definition 2 (Equilibrium)**.** *A group structure $(I, \mathbf{e}, \mathbf{p})$ is an **equilibrium**, if for almost all $w \in [\underline{w}, \overline{w}]$:*

$$U\left(w, e_{g(w)}(w), g(w), \mathcal{F}_{g(w)}\right) \geq U\left(w, k, \mathcal{F}_k\right), \quad \forall k \in A,$$

where g is an assignment function corresponding to I, $\{\mathcal{F}_k\}_{k \in A}$ are the social groups

[9]For example, in Bottan and Perez-Truglia (2022), preferences over income rank of medical graduates seem to arise from the effect on dating prospects. Complementarity is consistent with observed patterns of assortative mating (Kalmijn, 1998).

[10]Formally, for a given I, a corresponding $g : [\underline{w}, \overline{w}] \to A$ is as follows: $g(w) = \emptyset$, if $w \notin \bigcup_I W_i$; $g(w) = g(w') \in G$, if $\exists W_i \in I$ such that $w, w' \in W_i$; $g(w) \neq g(w')$ otherwise.

[11]As the label of a group is not payoff relevant, g is not uniquely defined for a given I, but any such g is equivalent in this context. Furthermore, the set G can be larger than the subset to which agents are assigned. But since joining an empty group yields the same payoff as not joining any, I treat such choices as identical. Given Assumption 3, this is without loss. Group choices in the action set A can be thought of as restricted to $\emptyset$ and the image of g.

generated by $(I, \mathbf{e}, \mathbf{p})$ *and* g*, and* $\mathbf{p}$ *describes their membership prices.*

It will be useful to vary the 'strength' of status concern to perform comparative statics and contrast outcomes against those where agents care only about quality or status. The following parametrised utility function is used for this purpose:

$$U_\alpha(w, k, \mathscr{F}_k) = e_k^*(w) \cdot \left[(1-\alpha)u(w, q_k) + \alpha v(w, r_k(w))\right] - c(e_k^*(w)) - p_k, \quad \alpha \in [0,1] \quad (2)$$

Agents are said to be *without status concern* if $\alpha = 0$; this is denoted by U_q. They have *only status concern* if $\alpha = 1$, which is denoted by U_r.

3 Equilibrium Groups

Laying the groundwork for the later analysis, this section characterises some fundamental equilibrium properties. These highlight the effects of status concern on any group structure, independent of how they arise. Example 1.1 illustrates the mechanics of the model and some of the subsequent results. As a key take away, it shows that if status concern is sufficiently strong, sorting cannot be achieved in equilibrium. While the set-up calls for a continuum of types, this is mostly done for mathematical convenience and most qualitative aspects do not hinge on this continuity assumption. The example transfers this to an analogous discrete setting.

Example 1.1. Suppose there are types $w \in \{1,2\}$, both equally likely in the population. Utility U_α is such that $u(w,q) = qw$, $v(w,r) = rw$, and $c(e) = e^2$. Group quality equals the average type, independent of e. Rank is as follows: if a group contains only type w, then $r(w) = 1/2$. If a group contains both types, then $r(1) = 1/4$, while $r(2) = 3/4$. Finally, a single agent joining a group of higher (lower) types obtains rank 0 (1).
Consider the segregated partition $I_{\text{seg}} = \{\{1\},\{2\}\}$, and assignment $g(1) = 1, g(2) = 2$. The corresponding social groups are homogeneous in type, with $q_1 = 1$ and $q_2 = 2$. Suppose membership prices are such that $p_1 = 0$ and $p_2 \geq 0$. Type $w = 1$ in $\mathscr{F}_1$ obtains $U_{1,1} = \frac{1}{4}(1-\frac{\alpha}{2})^2$, while $w = 2$ in $\mathscr{F}_2$ receives $U_{2,2} = \hat{U}_{2,2} - p_2 = (2-\frac{3}{2}\alpha)^2 - p_2$. A deviation by (a single) type 1 to $\mathscr{F}_2$ yields $U_{1,2} = \hat{U}_{1,2} - p_2 = \frac{1}{4}(2-2\alpha)^2 - p_2$, while a deviation by a type 2 yields $U_{2,1} = 1$. By setting $p_2 = \hat{U}_{2,2} - U_{2,1}$, incentive compatibility (IC) is satisfied as long as $\alpha \leq 2/3$ and $e_i(i) = e_i^*(i)$, $i \in \{1,2\}$. Accordingly, $(I_{\text{seg}}, \mathbf{e}, \mathbf{p})$ is an equilibrium. However, for $\alpha > 2/3$, we have $U_{2,1} > \hat{U}_{2,2}$, and thus $p_2 < 0$. While IC for type $w = 2$ is satisfied, we have $U_{2,1} - \hat{U}_{2,2} > U_{1,1} - \hat{U}_{1,2}$. This implies that for p_2, types $w = 1$ would strictly prefer membership in $\mathscr{F}_2$. If $\alpha > 2/3$, segregation is not an equilibrium. ⋄

3.1 Strictly Assortative Groups

Without crowding effects or returns to scale, sorting individuals into distinct groups is facilitated by assortativity. In the absence of status concern, complementarity and monotonic quality ensure that any interval partition can be implemented in equilibrium with uniform prices alone. Groups with members of higher type (and weakly higher engagement) have (weakly) higher quality (Assumption 4). Since higher types value quality more, we can find membership prices that deter lower types from joining. In a Tiebout setting, for example, people with a stronger preference for certain public goods sort into communities with a better provision of these, but also higher local taxes. Lemma 1 shows that this simple assortative matching logic, reminiscent of Becker (1973), holds true when agents have status concern. There is, however, a subtle difference: Lemma 1 rules out any equilibrium groups with equal quality. It allows us to strictly and equivalently order groups by price, quality, and member types. If we observe different groups in the presence of status concern, there necessarily is a benefit to sorting.

Lemma 1. *Suppose for preferences U, the group structure $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium with corresponding social groups $\{\mathscr{F}_k\}_{k=1}^{n}$. Then the following properties hold:*

(i) the support over types of every social group is convex and I is an interval partition of some $[w_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$,

(ii) for any two social groups $\mathscr{F}_l, \mathscr{F}_h \in \{\mathscr{F}_k\}$, $q_h > q_l$ or $q_l > q_h$,

(iii) for any two social groups $\mathscr{F}_l, \mathscr{F}_h \in \{\mathscr{F}_k\}$, $q_l < q_h \iff \overline{w}_l \leq \underline{w}_h \iff p_l < p_h$.

3.2 Limits to Sorting

How can individuals be sorted in equilibrium? In other words, can we find engagement levels and prices that make a given division of agents consistent with an equilibrium group structure? Following Lemma 1 (i), we can restrict our focus to those divisions that satisfy the necessary partition structure. Given individually optimal engagement levels, sorting with strict complementarities usually renders incentive compatibility (IC) only relevant for the 'cut-off type' between two groups. With status concern, however, achieving IC becomes more complex and might fail, as was demonstrated in Example 1.1. Lemma 2 formally shows that indifference of the cut-off type is necessary but no longer sufficient. There might be no equilibrium for a given I. Furthermore, this hinges only on prices, not engagement: Lemma 4 (Appendix A.1) establishes that for any interval partition I, there exists an incentive compatible $\mathbf{e}$, and corresponding social groups and group qualities. Nevertheless, there might be no prices that make the group choices incentive compatible. As a first key observation, status concern impedes and possibly prevents sorting (Proposition 1). It limits how homogeneous groups can be in type.

Consider the following recursive definition of prices:

$$p_1 = \begin{cases} \hat{U}(\underline{w}_1, 1, \mathscr{F}_1) - \underline{u} & \text{if } \underline{w}_1 > \underline{w} \\ p \leq \hat{U}(\underline{w}_1, 1, \mathscr{F}_1) - \underline{u} & \text{if } \underline{w}_1 = \underline{w}, \end{cases}$$

and for all $k > 1$:

$$p_k = p_{k-1} + \hat{U}(\underline{w}_k, k, \mathscr{F}_k) - \hat{U}(\underline{w}_k, k-1, \mathscr{F}_{k-1}) \tag{3}$$

Lemma 2. *Suppose I is an interval partition of some $W = [\underline{w}_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$. Without status concern, $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium if and only if $\mathbf{p}$ is as defined in* (3) *and $e_{g(w)}(w) = e^*_{g(w)}(w)$, for almost all $w \in W$. Under status concern, $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium only if $\mathbf{p}$ is as in* (3) *and $e_{g(w)}(w) = e^*_{g(w)}(w)$, for almost all $w \in W$.*

The proof is presented in Appendix A.2, but some key aspects are highlighted here to build intuition for subsequent results: in equilibrium, types at the cut-off between two groups must be indifferent between them. This uniquely pins down prices, except possibly p_1 (if $\underline{w}_1 = \underline{w}$, then this type can strictly prefer $\mathscr{F}_1$ to not joining a group). Prices have to satisfy (3). If only quality matters (U_q), complementarity in type and quality ensures IC for all other types and hence $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium. With status concern, however, IC can nevertheless fail. This has two reasons: (a) the price difference is insufficient to deter some types with lower status to join the higher quality group. And (b), which is more subtle, higher types might prefer the lower group. Prices in (3) balance the trade-off between rank quality for the cut-off type. They satisfy:

$$p_h - p_l = \underline{e}_h \cdot \left[u(\underline{w}_h, q_h) - \frac{\overline{e}_l}{\underline{e}_h} u(\underline{w}_h, q_l)\right] + \underline{e}_h \cdot \left[v(\underline{w}_h, 0) - \frac{\overline{e}_l}{\underline{e}_h} v(\underline{w}_h, 1)\right] - \left(c(\underline{e}_h) - c(\overline{e}_l)\right),$$

where $\underline{e}_h \equiv e_h(\underline{w}_h)$ and $\overline{e}_l \equiv e_l(\underline{w}_h)$. While lower types value quality less, they also have less status to lose since $v(w, r_l(w)) < v(w, 1)$. This might lead them to prefer the higher quality group. IC fails due to (a). Furthermore, for the cut-off type, higher quality in $\mathscr{F}_h$ might not fully compensate for the lower rank. The utility difference is negative; indifference thus requires $p_h - p_l < 0$. This leads again to (a). However, despite the single-crossing assumption, IC is not even ensured for all higher types. Some types might require an even greater reduction in p_h to accept the lower status in $\mathscr{F}_h$. IC then also fails because of (b), which plays an important role when considering price discrimination and transfers. This allows us to conclude that the set of possible equilibrium group structures is smaller under status concern. Proposition 1, and Corollaries 1.1 and 1.2 show how this affects the maximum number of groups in particular.

Proposition 1. *An equilibrium group structure* $(I, \mathbf{e}, \mathbf{p})$ *with* $I \neq \emptyset$ *exists. Under status concern (U), there is an upper bound* $N \geq 1$ *on the number of social groups in equilibrium. For preferences* U_r*, this upper-bound is 1. Without status concern (*U_q*), no such upper-bound exists.*

Corollary 1.1. *For preferences* U_α*, there exists* $\underline{\alpha} < 1$ *such that for all* $\alpha \in (\underline{\alpha}, 1]$*, in any equilibrium there is at most one social group.*

Corollary 1.2. *For preferences* U_q *and any interval partition I, there exists an equilibrium group structure* $(I, \mathbf{e}, \mathbf{p})$.

With status concern, sorting is limited in the sense that I cannot be arbitrarily fine. Recall that in Example 1.1, if status concern is sufficiently strong, equilibrium only allows for a single group. This insight extends to the general case: as $\alpha \to 1$, meaning individuals care only about rank, the maximum number of equilibrium groups approaches 1. Furthermore, this is not only true in the limit, but also for some $\alpha < 1$.[12] In other words, if status takes a predominant role in group membership, there can be only one group; sorting is prevented. In contrast, $\alpha = 0$ allows for any degree of sorting.

Corneo and Jeanne (1998, 1999) demonstrate how 'social segmentation' can affect positional concerns. The argument here highlights that there is also an effect in the other direction. Positional concerns limit segmentation. Benefits from sorting not only need to be positive, but also sufficiently large. Suppose, for example, a local school authority tries to separate students along different ability levels. If the quality of education is nevertheless similar across these schools, then such a separation cannot be achieved through (non-discriminating) tuition fees. Students that have a low rank in the lower quality school would strictly prefer the higher quality school if differences in tuition fees are small. If price differences are large, however, low ranked students in the higher quality school strictly prefer to switch and achieve a high rank. Depending on the quality differences, there might be no set of prices that balances both sides.

4 Provision of Groups

Rather than just asking which groups can exist, we might be interested in what groups a provider chooses to offer. A school committee might determine local school options

[12] Lemma OA1 in the Online Appendix further shows that if engagement is equalised across groups and types, there exists a least upper bound that is (weakly) monotonically decreasing in α.

and how to set entry barriers. Or a housing board might want to plan new residential developments. The analysis mainly addresses two extreme but instructive cases: the social planner problem looks at group provisions that maximise welfare, while the monopolist's problem addresses profit maximisation. This is then further contrasted to outcomes in a competitive market.

For this purpose, an additional stage is introduced. A provider first offers a group structure and agents then choose which group to join. The *equilibrium* notion remains essentially as before (Definition 2): the offered group structure needs to be incentive compatible. While there might be other incentive compatible group and engagement choices for a given set of prices,[13] attention is restricted to those consistent with the offer; the provider-preferred equilibrium. As the only explicit addition, the sum of payments is required to be non-negative, meaning any provider has to at least achieve budget balance. These equilibria are referred to as *equilibrium group provisions*. The *unconstrained maximum* refers to the provisions that maximise the provider's objective in the absence of any incentive constraints.

We can write the *planner problem* as the optimal choice of group assignment $g(w)$ (as in Definition 2), engagement levels $\mathbf{e}$, and membership prices $\mathbf{p}$:

$$\begin{aligned} \max_{g(w),\boldsymbol{e},\mathbf{p}} & \int \left[\hat{U}_\lambda(w, g(w), \mathscr{F}_{g(w)})\right] dF(w) \\ s.t. \quad & U_\alpha(w, e_{g(w)}(w), g(w), \mathscr{F}_{g(w)}) \geq U_\alpha(w, k, \mathscr{F}_k), \quad \forall k \in A, w \in [\underline{w}, \overline{w}], \\ & U_\alpha(w, g(w), \mathscr{F}_{g(w)}) \geq \underline{u}, \quad \forall w \in [\underline{w}, \overline{w}], \\ & \int p_{g(w)} dF(w) \geq 0, \end{aligned} \tag{4}$$

where $\lambda \in [0, 1]$, and any social group $\mathscr{F}_k$ is determined by g and $\mathbf{e}$. In case prices are not required to be uniform, the planner optimally chooses prices as function of verifiable characteristics. This is when the last inequality becomes particularly relevant: any transfers need to be budget balanced. U_λ reflects preferences that (potentially) consist of both the quality and the status component. If $\lambda > 0$, positional concerns enter the planner's objective. The planner aims to 'allocate' both quality and status efficiently. If $\lambda = \alpha$, the planner weights positional concerns just as individuals do. When referring to preferences U, U_r, and U_q, it is assumed that $\lambda = \alpha$. However, the setting allows for differences in their relative importance. For instance, if status is linked to a psychological rather than a tangible payoff, one could argue that welfare maximi-

[13] Even if several groups are offered, agents might coordinate on joining the one with the lowest price. Depending on the quality function, this could be IC. It seems, however, a plausible starting point to assume that a provider has the ability to resolve coordination issues. This thus neglects possible inefficiencies from coordination in favour of the more direct effects of status concern.

sation should ignore such aspects. In this case, $\lambda = 0 < \alpha$. Note that prices are not included under the integral ($\hat{U}$). They simply serve the purpose of maintaining incentive compatibility and do not affect welfare otherwise. For instance, this would be the case if payments are refunded as lump-sum transfers. It implicitly assumes sorting is costless; a baseline case that does not inherently bias against sorting.

The monopolist faces the same constraints but maximises revenue:

$$\begin{aligned} &\max_{g(w),\mathbf{e},\mathbf{p}} \int p_{g(w)} dF(w) \\ &s.t. \quad U_\alpha(w, e_{g(w)}(w), g(w), \mathscr{F}_{g(w)}) \geq U_\alpha(w, k, \mathscr{F}_k), \quad \forall k \in A, w \in [\underline{w}, \overline{w}] \\ &\qquad\ U_\alpha(w, g(w), \mathscr{F}_{g(w)}) \geq \underline{u} \quad \forall w \in [\underline{w}, \overline{w}], \end{aligned} \tag{5}$$

where again, any social group $\mathscr{F}_k$ is determined by g and $\mathbf{e}$.

Status concern affects the trade-offs that determine the 'optimality' of a group for either objective function. From the perspective of a single group, adding lower types has a possibly negative effect on quality but extends the benefits of membership to more individuals. Moreover, it raises the rank of higher types. In Example 1.1, extending a group of high types to include all types reduces quality from 2 to 3/2. But it also raises the rank of high types from 1/2 to 3/4, partially offsetting the impact of lower quality. The optimal group structure has to balance these forces within and across groups. For a social planner, these effects matter for all group members. A monopolist, however, is at least in the absence of price discrimination only concerned with the consequences for the lowest type; the one who determines the membership price. For a single group, the trade-off faced by a profit maximiser is thus essentially the same as in the absence of status concern. Nevertheless, status concern matters for the profitability of sorting as it affects price differences between groups, both directly through status differences at the cut-off, as well as indirectly through engagement and quality.

4.1 Status and Segregation

Whether allowing for more sorting by offering a more segregated (finer) group structure increases welfare is determined by the 'convexity' of the group quality. If splitting a group sufficiently increases the average quality across groups, such a split is welfare improving. For a monopolist, finer partitions also allow prices to be set more precisely according to types, thus facilitating higher profits. Without status concern, both incentives can lead to arbitrarily fine sorting. For example, if the quality of a group only depends on the lowest type - directly or through engagement - then refining any group structure increases welfare and revenue. As was shown in Proposition 1, status concern imposes a limit on the number of equilibrium groups and hence the degree of segregation. As is shown here, status concern also lowers a planner's and monopolist's incentive to segregate in the first place. Example 1.2 demonstrates this.

Example 1.2. The segregated partition $I_{\text{seg}} = \{\{1\},\{2\}\}$ in Ex.1.1 achieves welfare $\hat{U}_{\text{seg}} = \frac{1}{2}\hat{U}_{1,1} + \frac{1}{2}\hat{U}_{2,2} = \frac{1}{8}(1-\frac{\alpha}{2})^2 + \frac{1}{2}(2-\frac{3}{2}\alpha)^2$. Compare this to $I_{\text{int}} = \{\{1,2\}\}$. The average type and thus quality of the corresponding social group $\mathscr{F}_0$ equals $3/2$. Individual utilities are $\hat{U}_{1,0} = \frac{1}{4}(\frac{3}{2}-\frac{5}{4}\alpha)^2$ and $\hat{U}_{2,0} = (\frac{3}{2}-\frac{3}{4}\alpha)^2$. Welfare equals $\hat{U}_{\text{int}} = \frac{1}{8}(\frac{3}{2}-\frac{5}{4}\alpha)^2 + \frac{1}{2}(\frac{3}{2}-\frac{3}{4}\alpha)^2$. For $\alpha \leq 2/3$, segregation maximises welfare. For higher α, an integrated group is optimal. The same is true for revenue. If status concern is sufficiently strong, sorting can neither be achieved nor is it efficient. ◇

Intuitively, if ranks could be freely allocated and $\lambda > 0$, complementarity would lead a planner to assign high ranks to high types. Splitting any social group assigns almost all agents in the higher group a lower rank, and vice versa. If higher types value rank more, this leads to a drop in welfare. But even if all types value status equally (i.e., $v(w,r) = v(r)$), the strict complementarity between type and quality implies that higher types have a higher valuation for status through their choice of engagement (at least for $\alpha < 1$). The overall welfare effect of creating more homogeneous groups is less positive (more negative) under status concern. Generally speaking, segregation (potentially) helps to match quality and type efficiently but there is an accompanying loss from the mismatch between type and rank. As Proposition 2 shows, for sufficiently strong status concern (α high enough), the first-best partition consist of a single group, meaning $|I| = 1$.[14] This also holds if the planner merely attaches sufficient importance to status (λ high enough), independent of the individuals' actual status concern.

Status concern also weakens the incentive to segregate for a profit maximising provider. Suppose, for example, a private company is tasked with developing a housing project. If residents are concerned with their relative standing in the community, then the revenue from offering segregated communities is lower than if they only cared about quality aspects. The stronger the concern for status, the more beneficial it is to offer a more inclusive community. This, however, does not arise from the complementarity of type and rank but from the incentive constraint at the cut-off. Finer partitions allow for more price discrimination. But with status concern, higher types have an incentive to obtain a high rank in a lower quality group. Differences in membership prices need to be smaller. The revenue effect of segregation is less positive.

[14] Examining the benefits from sorting for all partitions that can be ordered by coarseness is less straightforward. Different utility levels lead to different engagement choices, affecting the benefits from sorting through $c(e)$ and q. Lemma OA2 in the Online Appendix shows that, abstracting from such effects, status concern makes offering finer group structures less beneficial to a planner and a monopolist.

Proposition 2. *Suppose preferences U_α are such that there is a strict complementarity between type and status. Then there exists $\underline{\alpha} < 1$, such that for all $\alpha \in (\underline{\alpha}, 1]$ and $\lambda = \alpha$, both the unconstrained and the equilibrium welfare- and profit-maximising group provisions do not allow for sorting ($|I| = 1$).*

Corollary 2.1. *There exists $\underline{\lambda} < 1$, such that for all $\lambda \in (\underline{\lambda}, 1]$, $\alpha \in [0,1]$, and strict complementarity between type and status, the unconstrained welfare-maximising provision does not allow for sorting.*

In Example 1.2, segregation fails to be implementable exactly at the point where it is no longer efficient. However, the limitations on sorting are not generally fully aligned with the welfare effects (Example 1.3). Combining Corollary 1.1 and Proposition 2 allows us to conclude that if status concern is sufficiently strong, sorting can neither be achieved, nor is it optimal. But the α- and λ- thresholds where sorting is prevented and where this becomes optimal generally differ. Inefficiency from an absence of sorting is thus an issue arising for intermediate levels of status concern.

Example 1.3. Suppose quality is equal to the lowest type within a group (assuming individual deviations have no effect on quality). This leaves q_1 and q_2, as well as the incentive constraints as before. But it lowers the quality of the integrated group to $q_0' = 1$. Welfare under I_{int} equals $U_{\text{int}}' = \frac{1}{8}(1 - \frac{3}{4}\alpha)^2 + \frac{1}{2}(1 + \frac{1}{4}\alpha)^2$. Segregation achieves higher welfare for $\alpha < \overline{\alpha} = 0.825...$, but cannot be achieved for $\alpha > 2/3$. For $\alpha \in (2/3, \overline{\alpha})$, the (unconstrained) welfare maximum cannot be provided in equilibrium. ◇

Proposition 3 and Example 1.4 further extend the analysis of the welfare effects regarding the strength of status concern. Holding constant λ, an increase in the individuals' status concern (α) reduces the possibility to sort. If quality is independent of engagement, meaning $q_k = \hat{q}_k$ if $F_k = \hat{F}_k$ (independent of $\mathbf{e}$), then this leads to a welfare loss (Proposition 3).

Proposition 3. *If quality is independent of engagement, then for any $\lambda \in [0,1)$, the welfare-maximising equilibrium for $\alpha = \lambda$ achieves (weakly) higher welfare than for any $\alpha' > \lambda$.*

If, however, engagement affects quality, it creates a (positive) externality. By neglecting such spillovers, individual equilibrium engagement is inefficiently low. This leaves the possibility for higher status concern to reduce inefficiencies, despite the constraints on sorting. Example 1.4 demonstrates how this can lead to a positive welfare effect: (i) high status concern can push engagement choices closer to the efficient level, and (ii), select for more efficient equilibria. Even if a planner attaches no weight to status ($\lambda = 0$), a positive level of status concern ($\alpha > 0$) can increase engagement and hence welfare. Effect (ii) arises when the feedback between quality and engagement allows for multiple equilibria. While the set-up allows the planner to select for equilibria (by the choice of $\mathbf{e}$ and $\mathbf{p}$), the example demonstrates how status concern can directly rule out an equilibrium with inefficiently low group quality.

As a key observation, if engagement affects quality, a planner can benefit from more intense status effects - whether status concern enters the planner objective or not. Appendix A.7 contains an in-depth discussion and provides sufficient conditions as well as a more detailed example for such positive welfare effects.

Example 1.4. Let $u(w,q) = \frac{1}{2}wq$, and $v(w,r) = 2rw$. Suppose quality is determined by average engagement, where $q_k = q_l = 1/2$ if $e_k < 1/4$, $q_k = q_m = 3/4$ if $1/4 \leq e_k < 2$, and $q_k = q_h = 2$ otherwise, with e_k the mean engagement in group $\mathscr{F}_k$. Let $e_l^*(w)$ be the optimal effort for type w in a group with quality q_l, and equivalently for q_m and q_h. At any $\lambda = \alpha$, segregation achieves higher welfare. But as before, for status concern sufficiently high ($\alpha > 5/13$), segregation cannot be achieved in equilibrium. A group of types $w = 1$ with $e \geq 1/4$ and quality q_m achieves higher welfare than with lower quality q_l and engagement $e_l^*(1)$. However, for $\alpha < 1/5$, this is not IC, since $e_m^*(1) < 1/4$. At $\alpha = \lambda = 0$, for instance, types can be efficiently segregated, but engagement is inefficiently low (for $w = 1$). As status concern increases, segregation with groups of quality q_m and q_h becomes feasible, and at $1/3 \leq \alpha \leq 5/13$, this is the only segregated equilibrium, since $e_l^*(1) \geq 1/4$, ruling-out the equilibrium with inefficiently low q_l. The positive welfare effect of intermediate levels of α extends to cases where planner and individuals attach different weights to status: welfare for $\lambda = 0$ and $1/5 \leq \alpha \leq 5/13$ is higher than for $\lambda = \alpha = 0$. ◇

4.2 Status and Social Exclusion

Another question pertains to the extent of participation. How does status affect the incentive of a social planner or monopolist to price agents out of the market? I refer to this more extreme version of segregation as *social exclusion*. For instance, tuition fees might be high enough to deter some people from acquiring higher education. Since the benefit from group membership in the absence of payments exceeds the stand-alone payoff (Assumption 3), social exclusion causes a welfare loss. It is shown here that excluding agents can nevertheless be a second-best outcome. Remarkably, a social planner might exclude more agents than a monopolist, in which case lower types benefit if the provider's objective is to maximise profit rather than welfare.

Suppose $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium with I a partition of an interval $[w_1, \overline{w}] \subset [\underline{w}, \overline{w}]$. We say $(I, \mathbf{e}, \mathbf{p})$ involves *social exclusion*, meaning not all types are assigned a group.

Proposition 4. *For every U and $\lambda = \alpha$, there exists a quality function q and distribution F, such that the welfare-maximising equilibrium group provision involves social exclusion. Social exclusion cannot be welfare-maximising if agents have preferences only over quality (U_q) or status (U_r).*

If there is no cost in providing groups, there is no direct welfare benefit in pricing agents out of the market. If a group structure involves social exclusion, offering the excluded individuals a separate group always increases welfare. Status concern, however, not only affects how fine a partition can be, but also which agents can be included. Social exclusion can be a constrained optimum. But this is only the case if welfare is determined by *both* quality and rank, either through $\alpha \in (0,1)$ or $\lambda \in (0,1)$. Social exclusion by a planner, at least in the absence of any cost of providing groups, is caused by the interaction of both types of peer effects.[15] As Corollary 4.1 makes explicit, if agents and planner care only about rank, the unique welfare-maximising group provision does not just consist of a single group (Proposition 2) but one that comprises all agents.

Corollary 4.1. *With preferences over status only (U_r) and $\lambda = \alpha$, there is a unique welfare-maximising equilibrium provision $(I, \mathbf{e}, \mathbf{p})$, with $I = \{[\underline{w}, \overline{w}]\}$. This equilibrium achieves the unconstrained welfare maximum.*

From a profit-maximising perspective, the effect of status concern on exclusion are twofold. The first effect adheres to the same logic as in the classic monopoly screening problem of Mussa and Rosen (1978): serving low types has a negative effect on the revenue from higher value types. With uniform prices, offering any lower quality group decreases the revenue from higher quality groups by the intra-group utility difference:

$$\Big(\overline{e}_k \cdot u(\overline{w}_k, q_k) - \underline{e}_k \cdot u(\underline{w}_k, q_k)\Big) + \Big(\overline{e}_k \cdot v(\overline{w}_k, 1) - \underline{e}_k \cdot v(\underline{w}_k, 0)\Big) - \Big(c(\overline{e}_k) - c(\underline{e}_k)\Big),$$

where $\overline{e}_k = e_k^*(\overline{w}_k)$ and $\underline{e}_k = e_k^*(\underline{w}_k)$. Since $v(w,1) - v(w,0) > 0$, status concern reinforces the effect. It makes the benefit from group membership more heterogeneous which reduces a monopolist's ability to extract surplus and thus increases the incentive to exclude agents.

This model also highlights a second trade-off: offering a separate, lower quality group might not be possible. In such cases, exclusion becomes a question of whether

[15] The comparison to preferences U_q is arguably sensitive to the assumption that groups can be provided costlessly. Otherwise, the welfare benefit from providing a lower quality group might be outweighed by the cost of providing it. Furthermore, it is well known in the Tiebout literature that with a discrete number of agents, 'integer problems' of providing optimal groups can arise (Conley and Konishi, 2002). This can render social exclusion constrained optimal with non-positional preferences. Proposition 4, however, highlights that with status concern, these issues run deeper. Exclusion persists even as the population becomes 'large' and the costs vanish.

or not to include more agents in the lowest quality group. Extending the membership base by itself increases revenue. However, as lower types have a lower willingness to pay and possibly a negative effect on quality, the membership price that can be charged decreases. This leaves the overall effect ambiguous. While extending the membership base is also beneficial for a welfare maximiser, any negative effect on quality is more pronounced. Quality changes are not just evaluated at the cut-off, but taking into account the effects on all members. The following result shows that this can lead to strictly more social exclusion when a planner serves the market - something that cannot be the case in the absence of status concern.

Corollary 4.2. *There exist U_α, q, and F, such that the welfare-maximising equilibrium group provision with $\lambda = \alpha \in (0,1)$ involves social exclusion while the profit-maximising one does not.*

4.3 Competitive Provisions

Instead of a monopolist, the market for social groups might be served by multiple providers; municipalities competing for inhabitants or universities trying to attract students, with the goal to maximise revenue. This most closely resembles the competition between jurisdictions as outlined in Tiebout (1956); a setting where communities attract residents from a heterogeneous population by differentiation in public good provision and tax levels. This section provides a brief analysis of some of the implications of such a competitive environment.

Given the continuum of players, the coordination aspect needs to be carefully considered as single-player deviations on the agents' side have no bearing on the aggregate outcome. Even if a provider offers a potentially Pareto-improving group structure, joining such a group alone is not beneficial from the perspective of any individual. This would prevent competition between providers. On the other hand, allowing for any joint deviation, independent of whether or not the resulting group structure is proof to deviations itself, leads to well-known existence issues (Bewley, 1981). I thus restrict attention to deviations that weakly benefit all agents (Pareto-improving from the agents' perspective), rather than just some group. This closely reflects the equilibrium definition in Greenberg (1983), that deals with the analogous issue in the discrete case, partly in response to the critique of Bewley (1981).

The set-up is modified as follows: there is a large (countable) set of potential providers or firms, denoted by Z, that maximise profits. Each firm can offer one or a set of social groups (defined by I and e) and freely set (uniform) membership prices or subsidies ($p < 0$), as long as they achieve at least 0 profit. Any offer, whether this is from one or a combination of firms, that satisfies IC on the agents' side and the non-negative profit condition on the firms' side is called a *competitive group provision.* More precisely, if for a given group structure, we can find a function (ζ) that assigns groups to firms such that no firm makes a loss, then it is, at least in principle, consistent with profit-maximising firms serving this market. In line with the notion of joint deviations, such a group structure cannot be an equilibrium if any other provider could make a competing offer that Pareto-dominates it from the agents' perspective. For example, if an identical partition could be offered at lower prices, this would make all agents strictly better off. Following the usual Bertrand reasoning, such Pareto-superior provisions would always be offered, since it allows some firm(s) to capture a larger market share.

Definition 3 (Competitive Equilibrium)**.** *A group provision $(I, \mathbf{e}, \mathbf{p})$ is* ***competitive****, if it is an equilibrium, and there exists a function $\zeta : A \mapsto Z$, such that for all $z \in Z$:*

$$\int_{\underline{w}}^{\overline{w}} p_{g(w)} \cdot \mathbb{1}_{\{\zeta(g(w))=z\}}\, dF(w) \geq 0.$$

It is a ***competitive equilibrium*** *if there is no other competitive group provision that Pareto-dominates it from the agents' perspective.*

Proposition 5. *Suppose the group structure $(I, \mathbf{e}, \mathbf{p})$ is a competitive equilibrium. Then the following properties hold:*

(i) if I involves no social exclusion then $p_1 \leq 0$, with the inequality strict if $|I| > 1$, and all firms make 0 profit;

(ii) if I involves social exclusion, then there exists no equilibrium group structure $(I', \mathbf{e}', \mathbf{p}')$, with interval partition $I' = I \cup [w_0, \underline{w}_1]$. All firms providing groups make positive profits;

(iii) there is no other distinct equilibrium group structure $(I, \mathbf{e}', \mathbf{p}')$ with $e'_k(w) \geq e_k(w)$, for all $k \in A$ and $w \in [\underline{w}_1, \overline{w}]$.

The effects of competition hinge on whether or not a group structure involves social exclusion. For full-participation group structures, competition has the predictable effect of lowering prices relative to a monopoly provider. If there is more than one social group, at least the members of the lowest quality group receive a subsidy instead of paying for membership. Furthermore, no set of individuals is excluded if they could be offered a separate group in equilibrium. Competition thus reduces the set of provisions involving social exclusion. As indicated by *(ii)*, however, this set is not necessarily empty. With social exclusion, firms cannot compete over partitions and prices separately. If agents are excluded, prices are uniquely pinned down (Lemma 2). Competition cannot reduce prices without altering the partition at the same time. While this would also be the case without status concern, this case is irrelevant with preferences over quality (or status) alone, as such a provision is necessarily Pareto-dominated.[16] With preferences U, social exclusion can be Pareto-efficient among the competitive provisions. Competition does not necessarily reduce membership payments or extend the benefits of group membership to more individuals - a result specific to populations with preferences over both quality and status. Example 2.1 demonstrates that competitive equilibria can, in fact, involve *more* social exclusion than a monopoly provision. Finally, *(iii)* is another familiar effect of competition: it rules out the underprovision of quality, as is possible in the monopoly case. If there are multiple equilibria for a given partition, competition selects the one with the highest engagement and hence group quality.

Example 2.1. Suppose types are distributed according to a (truncated) Pareto distribution over $[1,3]$ with shape parameter s. For simplicity, abstract from engagement choices and suppose $e_k^*(w) = 1$, for all $k \in A$ and $w \in [\underline{w}, \overline{w}]$, and $c(1) = 0$. Utility is given by $u(w,q) = qw$ and $v(w,r) = (2r-1)w$, with $\underline{u} = 0$. The quality of a group k is defined as:

$$q_k = \begin{cases} q_h & \text{if } \underline{w}_k \geq 2 \\ 2 & \text{otherwise.} \end{cases}$$

Consider the partitions $I_{\text{int}} = \{[1,3]\}$, $I_{\text{exc}} = \{[2,3]\}$, and $I_{\text{seg}} = \{[1,2],[2,3]\}$. Denote the corresponding social group with support over $[1,3]$ by $\mathscr{F}_0$, with support over $[1,2]$ by $\mathscr{F}_1$, and with support over $[2,3]$ by $\mathscr{F}_2$. Group qualities are $q_0 = q_1 = 2$ and $q_2 = q_h$. For $q_h = 4$, $u(2,q_1) + v(2,1) = u(2,q_2) + v(2,0)$, meaning there is no price vector $\mathbf{p}$ such that I_{seg} can be achieved in equilibrium. IC at the cut-off requires $p_h = 0$, which violates IC for all $w < 2$. Suppose inequality is relatively low with $s = 1$. Then $u(1,q_1) + v(1,0) < \left[1 - F(2)\right]\left(u(2,q_2) + v(2,0)\right)$, which implies profits are maximised under I_{exc} (with $p_2 = u(2,q_h) + v(2,0)$). As inequality increases ($s = 2$), we have $u(1,q_1) + v(1,0) > \left[1 - F(2)\right]\left(u(2,q_2) + v(2,0)\right)$, and so profits are maximised under I_{int}. Furthermore, welfare is equally strictly higher under I_{int}. Note that $u(3,q_2) + v(3,1) - p_2 = u(3,q_1) + v(3,1)$, meaning type $\overline{w}$ achieves the same utility under I_{int} and I_{exc}. As all types below prefer I_{int}, integration is a competitive equilibrium. However, with $q_h = 4 + \epsilon$ for small $\epsilon > 0$, type $\overline{w}$ achieves strictly higher utility with I_{exc}. As all other

[16]For preferences U_q, a firm can offer a separate group to the excluded agents while leaving all other social groups unaltered. This increases utility of the members of the additional group and lowers prices for everyone else. For U_r, a single group involving all agents Pareto-dominates all others.

constraints remain qualitatively unaltered, I_{exc} can be achieved in a competitive equilibrium, while it is neither profit- nor welfare-maximising. ⋄

5 Restoring Efficient Sorting

This section explores two approaches how to restore a provider's ability to sort individuals and remedy resulting inefficiencies: price discrimination and screening.

5.1 Price Discrimination

If individuals have preferences over quality or status only, uniform prices are sufficient to implement the welfare-maximising group structure. Price discrimination only benefits a profit maximiser. As is argued here, however, if agents have preferences over quality *and* status, price discrimination can serve a purpose besides the extraction of surplus and might be present even if a social planner serves the market. Moreover, the qualitative effects of price-discrimination on sorting are distinctively different with status concern. Following the argument of Schelling (1971) and Arrow (1998), if agents have preferences over group composition (but not ordinal rank), then the absence of price discrimination can cause (inefficient) segregation.[17] In contrast, it is shown here that if preferences include status concern, price discrimination can remedy inefficient *integration* and thus potentially lead to more segregation, not less (Example 3.1).[18]

An obvious benchmark is to allow prices to be *type–dependent*, i.e., functions of w, denoted by $\mathbf{p}^w$. As Proposition 6 shows, this restores the ability to implement any positive sorting. It requires, however, that types are public information which can be used to tailor prices. In the context of tuition fees, this degree of price discrimination might not be feasible and potentially illegal. Moreover, due to the more complex incentive constraints, some group structures can require prices to bet set prohibitively high for types higher and lower than desired. Tuition fees would have to vary non-monotonically with ability within the same institution; a property unlikely to be implementable in practice. A more practicable approach with a lower informational requirement is to allow prices to adjust to quality and status in each group by being *rank-dependent* ($\mathbf{p}^r$). For example, in educational settings information about

[17] This is exemplified by results in Board (2009), showing that a monopolist offers overly fine sorting. Ellickson et al. (1999) demonstrate that price discrimination can help to integrate groups and reduce such 'over-sorting'.

[18] This resembles findings in the literature on clubs and Tiebout-sorting, where - albeit for different reasons - price discrimination and/or restrictions to free-mobility can be required to implement some desirable club configurations (Scotchmer (2005), Greenberg (1983)) and deal with issues of existence of equilibria (Bewley, 1981).

in-group rank, e.g. relative performance, is arguably more readily observable.

Proposition 6. *Suppose I is an interval partition of $[\underline{w}_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$. There exist type-dependent prices $\mathbf{p}^w$ and $\mathbf{e}$ such that $(I, \mathbf{e}, \mathbf{p}^w)$ is an equilibrium provision. There also exist rank-dependent prices $\mathbf{p}^r$ such that $(I, \mathbf{e}, \mathbf{p}^r)$ is an equilibrium provision if preferences satisfy $\frac{\partial^2}{\partial w \partial r} v(w,r) = 0$.*

A key point of Proposition 6 is that when status concern does not directly vary with type, rank-dependent pricing is sufficient to implement any interval partition (with a suitable $\mathbf{e}$). Suppose (hypothetically) that there is no complementarity at all between type and rank. Two-part membership prices consisting of a group- and a rank-dependent component could then 'neutralise' positional concerns and maintain IC. For example, if $v(w,r) = (r - \frac{1}{2})$, then a provider can charge a rank-based part $(r - \frac{1}{2})$, for any $r > \frac{1}{2}$, and offer an equal reduction to the 'corresponding' rank $1 - r$. With these balanced transfers, status does not create any incentive to deviate. Any interval partition can then be achieved in equilibrium with an additional group based (uniform) price component. Since higher types value quality more, however, they choose a higher engagement level for any given rank. This creates a strict complementarity between type and rank. Yet, as Proposition 6 shows, if $\frac{\partial^2}{\partial w \partial r} v(w,r) = 0$, prices can nevertheless be varied with rank so as to eliminate any restrictions on the degree of sorting. If status concern varies directly with type, however, such transfers are not always feasible, unless stronger assumptions are made. This is discussed more formally in Appendix A.4, where sufficient conditions are provided. Example 3.1 demonstrates a case where rank-based price discrimination can indeed restore (efficient) sorting despite complementarities between type and status, while Example 3.2 presents a counter-example.

Example 3.1. Suppose types are distributed uniformly over $[1,2]$, utility is given by $u(w,q) = qw^2$, $v(w,r) = rw^2$, $c(e) = e^2$, $\underline{u} = 0$, and the quality of a social group $\mathscr{F}_k$ depends on its lowest type as follows:

$$q_k = \begin{cases} 1.4 & \text{if } \underline{w}_k \geq 1.75 \\ 1 & \text{otherwise.} \end{cases}$$

Welfare is maximised with $I_{\text{seg}} = \{[1, 1.75], [1.75, 2]\}$. Since $\hat{U}(1.75, 1, \mathscr{F}_1) > \hat{U}(1.75, 2, \mathscr{F}_2)$, though, there are no uniform prices that achieve IC. The constrained welfare-maximising equilibrium partition is $I_{\text{int}} = \{[1,2]\}$. Consider instead the following rank-based prices for I_{seg}:

$$p_1(r) = \hat{U}(w_1(r), 1, \mathscr{F}_1) \qquad p_2(r) = p_1(1) + \hat{U}(w_2(r), 2, \mathscr{F}_2) - \hat{U}(w_2(r), 1, \mathscr{F}_1),$$

where $w_1(r) = 1 + \frac{3}{4}r$ and $w_2(r) = 1.75 + \frac{1}{4}r$ (the inverse of $r(w)$ within each group). These prices extract all surplus from members of $\mathscr{F}_1$ and leave just enough to members of $\mathscr{F}_2$ to compensate for the incentive to deviate. Furthermore, (almost) all types in $\mathscr{F}_1$ strictly prefer their group to $\mathscr{F}_2$. Since $p_1(1) = \hat{U}(r_2^{-1}(0), 1, \mathscr{F}_1)$, it is easily verified that prices, and thus revenue, are positive. The corresponding equilibrium group provision achieves higher welfare (and revenue) than one with I_{int}. ◇

As is the case in Example 3.1, efficient segregation can require that some members of a lower quality group pay a higher price than some types in a higher quality group. Consider a two-tiered education system, segregated by ability. If the relative position within their cohort affects students' payoffs, this necessitate lower tuition fees, or even stipends, for students to accept a low rank in the higher tier instead of a high rank in the lower tier. If payments can be tailored to an individual's relative position in their cohort, IC and budget balance can both be satisfied if the required subsidies are small enough. Price discrimination and transfers can thus serve to maintain segregation rather than integration. In fact, while transfers have a redistributive purpose without status concern, here they can reinforce inequalities (Online Appendix, Section 2).

Achieving segregation can, however, require subsidies not just within but also across groups. In Example 3.2, all members of the lowest quality group would need to be subsidised. Even for the welfare maximising group structure, these are not necessarily budget balanced when there is a complementary between type and status. While complementarity between type and quality facilitates sorting, any complementarity between type and status - even if less pronounced (Assumption 2) - obstructs it. Status concern then leads to inefficient integration even with rank-based prices. A provider might fail to implement the optimal group structure without additional screening mechanisms. In the case of university entrance, for example, tuition fees are rarely used as the only mechanism to select candidates, despite possible complementarities between educational quality and ability.

Example 3.2. Suppose the quality function from Example 3.1 is modified as follows:

$$q_k = \begin{cases} 1.4 & \text{if } \underline{w}_k \geq 1.75 \\ 1.25 & \text{if } \underline{w}_k \in [1.25, 1.75) \\ 1 & \text{otherwise.} \end{cases}$$

Welfare is maximised with $I_{\text{seg}} = \{[1, 1.25], [1.25, 1.75], [1.75, 2]\}$. The following prices extract the maximum surplus such that downward IC is satisfied:

$$\begin{aligned} p_1(r) = &\ \hat{U}(w_1(r), 1, \mathscr{F}_1) \\ p_2(r) = &\ p_1(1) + \hat{U}(w_2(r), 2, \mathscr{F}_2) - \hat{U}(w_2(r), 1, \mathscr{F}_1) \\ p_3(r) = &\ p_2(1) + \hat{U}(w_3(r), 3, \mathscr{F}_3) - \hat{U}(w_3(r), 2, \mathscr{F}_2). \end{aligned}$$

However, with these prices, all types in $[1, 1.75)$ would strictly prefer membership in $\mathscr{F}_3$. Consider, in addition, the following subsidies:

$$s_1(r) = \hat{U}(w_1(r), 3, \mathscr{F}_3) \qquad s_2(r) = \hat{U}(w_2(r), 3, \mathscr{F}_3) - \hat{U}(w_2(r), 1, \mathscr{F}_1),$$

while $s_3(r) = 0$. These are the minimum subsidies that ensure upward IC (while maintaining downward IC). For prices $\hat{p}_k(r) = p_k(r) - s_k(r)$, the group structure $(I_{\text{seg}}, \mathbf{e}, \hat{\mathbf{p}}^r)$ is an equilibrium. However, $\sum_{i=1}^{3}(F(\overline{w}_i) - F(\underline{w}_i)) \int_0^1 \hat{p}_i(r)dr < 0$, meaning budget balance fails. I_{seg} cannot be implemented as an equilibrium provision. ◇

5.2 Regulating Engagement

A natural way to include screening in this model is to allow a provider to regulate engagement choices. For instance, a university can make attendance compulsory, or a club can limit the use of its facilities. Even though $c(e)$ is assumed to be identical across types, restricting engagement can take the role of a screening device since utility from group membership, and hence marginal benefit from e, is increasing in type. Depending on the constraint, this attenuates or exacerbate the effects from social interactions. It allows a provider to widen the utility (and possibly quality) difference between groups, lower the incentive to deviate, and enable sorting. Formally, a provider can limit engagement choices in any group k to $e^*_k(w) \leq \overline{e}_k$ and/or $e^*_k(w) \geq \underline{e}_k$. Denote the restricted engagement vector by $\overline{\boldsymbol{e}}$. The equilibrium definition remains as before: $(I, \overline{\boldsymbol{e}}, \mathbf{p})$ is an equilibrium provision if group assignments are IC given the engagement limit, and the provider achieves at least 0 profit.

Independent of e, status concern exacerbates intra-group utility differences which affect membership prices and the incentives to deviate. But the extent to which individuals engage with the group simultaneously affects their benefits from quality and status. As Proposition 7 shows, sufficiently limiting e lowers intra-group differences enough to fully restore the ability of a provider to sort individuals with uniform prices.

Proposition 7. *Suppose quality is strictly monotonic in type. Then for any interval partition I of some $[w_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$, there exist $\mathbf{p}$ and $\overline{\boldsymbol{e}}$ such that $(I, \overline{\boldsymbol{e}}, \mathbf{p})$ is an equilibrium group provision.*

As an immediate consequence, introducing engagement limits eliminates the need for a planner to exclude agents. Adding some $\mathscr{F}_0$ that contains all excluded types to a given group provision with social exclusion increases welfare for all $\overline{e}_0 > 0$. But a sufficiently low $\overline{e}_0$ makes $\mathscr{F}_0$ unattractive enough to not violate downward IC. At the same time, it reduces the intra-group utility differences, which allows for a p_1 that ensures upward IC. More than just increasing welfare, this also constitutes a Pareto-improvement. Screening can benefit all individuals in this setting.

Corollary 7.1. *The welfare-maximising equilibrium group provision* $(I, \overline{\mathbf{e}}, \mathbf{p})$ *involves no social exclusion.*

While engagement upper-bounds are sufficient to restore sorting, welfare can be further increased by simultaneously allowing for engagement lower-bounds, i.e., an additional constraint $e_i(w) \geq \underline{e}_i$. Consider a setting where sorting agents in two groups, $\mathscr{F}_1$ and $\mathscr{F}_2$, is optimal, but cannot be achieved with uniform prices. As follows from Proposition 7, there exists an engagement limit (on the lower quality group) that allows for sorting. Any such limit, however, distorts choices and thus negatively affects welfare. An additional lower-bound on engagement equal to $e_2(\underline{w}_2)$ is non-binding for the intended members of $\mathscr{F}_2$, but binding for all lower types. This reduces the incentive to deviate, which allows for relaxing $\overline{e}_1$ without causing any distortions for members of $\mathscr{F}_2$. Interestingly, in some cases welfare can be further increased by imposing a binding lower-bound on the higher quality group - even if e does not affect quality (Example 3.3). Splitting the screening burden across groups by distorting engagement choices of higher and lower types can be more beneficial than distorting choices of lower types alone.

Example 3.3. Suppose type and utility are as in Example 3.1 and the quality of a group k depends on its lowest type as follows:

$$q_k = \begin{cases} q_h & \text{if } \underline{w}_k \geq 1.75 \\ 1 & \text{otherwise.} \end{cases}$$

First, consider the previous case with $q_h = 1.4$. Recall that segregation maximises welfare but there exist no uniform prices that make segregation IC. By imposing an upper bound $\overline{e}_1 < e_1^*(\overline{w}_1)$, IC can be satisfied with uniform prices. Welfare can be further increased by imposing an additional lower bound $\underline{e}_2 = e_2^*(\underline{w}_2)$ for $\mathscr{F}_2$. However, in both cases welfare of the integrated group exceeds welfare from segregation (see Table 1). Sorting can be achieved but is no longer efficient. Consider instead $q_h = 1.75$. There exists $\overline{e}_1 < e_1^*(\overline{w}_1)$ such that IC is satisfied with uniform prices and sorting remains welfare maximising. Again, imposing $\underline{e}_2 = e_2^*(\underline{w}_2)$ increases welfare. However, there exists a lower-bound $\underline{e}_2 > e_2^*(\underline{w}_2)$ that further increases utility. Under status concern, distorting choices in *both* low- and high-type groups can increase welfare. ◇

quality		engagement limits		welfare ($\sum \hat{U}$)
$q_{\text{int}} = 1$		-	-	4.54
$q_1 = 1$	$q_2 = 1.4$	$\underline{e}_1 = 0.82$	-	4.40
		$\overline{e}_1 = 0.88$	$\underline{e}_2 = e_2(\underline{w}_2)$	4.47
$q_1 = 1$	$q_2 = 1.75$	$\overline{e}_1 = 1.33$	-	6.01
		$\overline{e}_1 = 1.48$	$\underline{e}_2 = e_2(\underline{w}_2)$	6.09
		$\overline{e}_1 = 1.55$	$\underline{e}_2 = 3.00$	6.12

Table 1: Welfare for different engagement limits and group qualities (Ex. 3.3). For $q_1 = 1, q_2 = 1.75$, distorting engagement in both groups achieves higher welfare than some $\overline{e}_1$ alone.

6 Discussion

Inequality. Individuals can only be segregated into different group if there is a sufficient benefit to it (Sections 3.1 and 3.2). Even if segregating agents into several groups is a Pareto-improvement, sorting might not be incentive compatible, leading to inefficient integration and/or social exclusion. These instances are inherently linked to the inequality in the distribution of types in the population. Suppose, for instance, quality is determined by the mean-type in a group. Then there always exists a distribution with low-enough variance such that individuals cannot be split into two or more groups (Proposition 10, Appendix A.6). Independent of the exact utility function, the highest and lowest type in the lower quality group would value quality almost identically. Their benefit from group membership, however, varies due to the difference in rank within their group. Consequently, there are no (uniform) prices that make segregation incentive compatible. The only equilibrium outcome is a single group, while without status concern, any interval partition could be achieved. In a network environment with conspicuous consumption, Bramoullé and Ghiglino (2022) find that loss-aversion can lead to conformism, i.e., individuals consuming identical amounts of the conspicuous consumption good, if income inequality is low enough. Status concern similarly forces conformism in group choices if types are sufficiently homogeneous. For such type-distributions, individuals differentiate themselves through intra-group status rather than group affiliation.

Redistribution. Status concern allows for an additional perspective on the benefits and the political support for redistribution. Price discrimination and transfers (i.e., redistribution of some payments) matter for whether a group structure can be implemented in equilibrium. Welfare gains from redistribution arise directly from the effect on sorting. Rather than reducing inequality, such redistribution can lead to more heterogeneous equilibrium outcomes than would be possible otherwise. This is examined in detail in the Online Appendix. In close analogy to Levy and Razin (2015), it demonstrates how transfers that are strictly necessary for sorting can reduce the set of individuals in a society in favour of full redistribution and give sorting large majority support. Welfare-increasing sorting can require that its benefits are spread further in the population. In Example 3.2, the welfare-maximising group structure consists of 3 groups. IC requires a subsidy for members of the two lower quality groups.[19] The benefit from a more exclusive group obtained by higher-types needs to be shared across the population to achieve sorting in the first place. Support for such transfers may then not necessarily be motivated by an objective to reduce inequality, but aim to maintain a certain level of stratification. Positional concerns can thus help to explain why some transfers can be found even in stratified environments, why they find support even from individuals that significantly benefit from sorting, and why they not always target just the poorest (Acemoglu et al., 2015; Brady and Bostic, 2015).

Local vs global status. Instead of group quality, q can equally be seen as the prestige or status associated with group membership itself. Consistent with the assumptions, q could, for instance, be determined by the average rank of group members in the population distribution. Individuals thus obtain utility from their relative position within their group (local status), as well as their group affiliation which visible outside of their group (global status). With this shift in interpretation, the model lends itself to analyzing changes over time in the importance of social comparisons, their effects on sorting, and their welfare implications. As a specific example, it is well-documented that social networking websites have changed the nature and extent of social comparisons (Verduyn et al., 2020; Kross et al., 2021). Langtry (2023) argues that this can offer an explanation for increases in positive sorting and inequality observed in the labour market (Card et al., 2013; Song et al., 2018), since social comparisons with co-workers become relatively less important. This model offers a complementary perspective that extends to other domains: with preferences U_α, a decrease in α describes a relative decrease in the importance of intra-group comparisons. If a technological change facilitates global status comparisons, thus decreasing α, incentive compati-

[19]In the particular example, the necessary subsidies are not budget balanced. However, increasing q_3 to $q_3' = 2$ would allow for budget balance while still requiring redistribution.

bility constraints are relaxed, while simultaneously increasing the benefits of sorting. Membership in a high-status group, such as a degree from an elite university, becomes more visible and hence more important than the status within that group. This allows an increase in segregation in the population (i.e., a finer equilibrium group structure) and can thus increase overall inequality. Interestingly, despite increasing the set of equilibrium group structures, such a reduction in local status concern can lead to an inefficient group provision. For a sufficiently high α, the absence of sorting is not just the only equilibrium outcome but also a welfare maximum. As shown in Example 1.3, inefficiencies arise for intermediate levels of local status concern. Furthermore, lower status concern can lead to inefficiently low group engagement and quality (Example 1.4). Technological changes facilitating wider social comparisons might thus affect the welfare-maximising degree of sorting, its attainability, and its provision across different market structures.

Status concern and welfare. Positional concerns can cause significant welfare losses: they distort consumption and savings decisions, particularly when status is signalled through conspicuous consumption (Frank, 2005; Hopkins and Kornienko, 2004). They can also lead individuals to actively lower other players' payoffs and destroy surplus (Zizzo and Oswald, 2001). On the other hand, rank considerations can be crucial in promoting effort, at least if carefully induced through the distribution of prizes in tournaments (Moldovanu and Sela, 2001). As follows from the analysis, both of these aspects are present if the prime consideration is the efficient sorting of individuals. Status concern restricts the number and type of groups that can be offered, which can cause a welfare loss. This applies not just if utility from status is not reflected in the welfare criterion ($\lambda = 0$), but also if the planner and the individuals attach equal weight to status ($\lambda = \alpha$). At the same time, status concern can lead individuals to engage more with their group, enhancing positive spillovers. This allows for an overall positive welfare effect - even if the planner attaches no weight to status (Appendix A.7). When the benefits from social interactions depend on the peers as well as the intensity of these interactions, a social planner might want to induce or enhance positional concerns, even if they are not directly reflected in the planner's objective function.

A Appendix: Proofs and Additional Results

A.1 Additional Results (Section 3)

Lemma 3 shows that with quadratic engagement cost, single-crossing (Assumption 2) of $u(w, q) + v(w, r)$ extends to $\hat{U}$.

Lemma 3. *Suppose* $c(e) = a \cdot e^2$*. If* $w_2 > w_1$*,* $u(w_1, q_h) + v(w_1, r_h) - \big(u(w_1, q_l) + v(w_1, r_l)\big) = \delta > 0$*, and* $u(w_2, q_h) + v(w_2, r_h) - \big(u(w_2, q_l) + v(w_2, r_l)\big) \geq \delta$*, then*

$$\begin{aligned} &e_h^*(w_2) \cdot \big[u(w_2, q_h) + v(w_2, r_h)\big] - c(e_h^*(w_2)) - \Big(e_l^*(w_2) \cdot \big[u(w_2, q_l) + v(w_2, r_l)\big] - c(e_l^*(w_2))\Big) \\ >&e_h^*(w_1) \cdot \big[u(w_1, q_h) + v(w_1, r_h)\big] - c(e_h^*(w_1)) - \Big(e_l^*(w_1) \cdot \big[u(w_1, q_l) + v(w_1, r_l)\big] - c(e_l^*(w_1))\Big). \end{aligned}$$

Proof. Let $\overline{b}_i \equiv u(w_i, q_h) + v(w_i, r_h)$ and $\underline{b}_i \equiv u(w_i, q_l) + v(w_i, r_l)$, $i \in \{1, 2\}$. We can then rewrite the inequality as:

$$\begin{aligned} &e_h^*(w_2)\overline{b}_2 - c(e_h^*(w_2)) - e_l^*(w_2)\underline{b}_2 + c(e_l^*(w_2)) \\ >&e_h^*(w_1)\overline{b}_1 - c(e_h^*(w_1)) - e_l^*(w_1)\underline{b}_1 + c(e_l^*(w_1)). \end{aligned}$$

Optimality requires that $e_h^*(w_i) = \frac{\overline{b}_i}{2a}$, and accordingly $c(e_h^*(w_i)) = \frac{\overline{b}_i^2}{4a^2}$. We can rearrange the previous inequality to $\frac{\overline{b}_2^2}{4a} - \frac{\underline{b}_2^2}{4a} > \frac{\overline{b}_1^2}{4a} - \frac{\underline{b}_1^2}{4a}$, and hence:

$$(\overline{b}_2 - \underline{b}_2)(\overline{b}_2 + \underline{b}_2) > (\overline{b}_1 - \underline{b}_1)(\overline{b}_1 + \underline{b}_1). \tag{6}$$

But since $w_2 > w_1$, it follows from Assumption 1 that $\overline{b}_2 > \overline{b}_1$ and $\underline{b}_2 > \underline{b}_1$. Furthermore, it follows from the premise that $\overline{b}_2 - \underline{b}_2 > \overline{b}_1 - \underline{b}_1$. This means (6) is satisfied, as required. □

Lemma 4 establishes that for any interval partition I, we can find corresponding, incentive compatible engagement choices $e_{g(w)}^*(w)$ and hence social groups and group qualities. Accordingly, if for such an I there exists no equilibrium $(I, \mathbf{e}, \mathbf{p})$, then this hinges on $\mathbf{p}$.

Lemma 4. *Let* I *be an interval partition of some* $[w_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$ *and* g *the corresponding assignment function. Then there exist* $\mathbf{e}$ *and corresponding social groups* $\{\mathscr{F}_k\}_{k=1}^{|I|}$*, such that* $e_{g(w)}(w) = e_{g(w)}^*(w)$*, for almost all* $w \in [w_1, \overline{w}]$*.*

Proof. Let $\{W_k\}_{k=1}^{|I|}$ be the intervals in I, with $W_1 = [\underline{w}_1, \overline{w}_1]$ the one with the lowest types. Suppose $\big(q_k^{(i)}\big)_{i=0}^{\infty}$ denotes an infinite sequence of real numbers with $q_1^{(0)} = \underline{q}$ and the remainder constructed as follows:

Define for each $w \in W_1$ and $i \in \mathbb{N}$, $i > 1$:

$$e_1^{(i)}(w) = \arg\max_e \, e \cdot \big[u(w, q_1^{(i-1)}) + v(w, r_1(w))\big] - c(e). \tag{7}$$

where $r_1(w)$ is a type w's rank according to F_1, with F_1 determined by F and I. It follows from Berge's Maximum Theorem and continuity of u and v that $e_1^{(i)}(w)$ is continuous in w. Let $\mathscr{F}_1^{(i)}$ be the social group corresponding to W_1 and $e_1^{(i)}(w)$ with its quality denoted by $\tilde{q}$. Set $q_1^{(i)} = \tilde{q}$. Clearly, $q_1^{(1)} \geq q_1^{(0)} \geq \underline{q}$. If $q_1^{(1)} = q_1^{(0)}$, then for all $i > 1$, $q_1^i = q_1^{(1)} = \lim_{i \to \infty} q_1^{(i)}$. If instead $q_1^{(1)} > q_1^{(0)}$, then $e_1^{(2)}(w) > e_1^{(1)}(w)$ for all $w \in W_1$, since $u(w, q)$ is strictly increasing in q and $c(e)$ is continuous. Monotonic quality (Assumption 4) ensures $q_1^{(2)} \geq q_1^{(1)}$. We can construct the sequence $\big(q_1^{(i)}\big)_{i=0}^{\infty}$ accordingly.

By the previous arguments, $q_1^{(i+1)} \geq q_1^{(i)}$, for all $i \in \mathbb{N}$. Furthermore, since quality is bounded, $q_1^{(i)} \leq \overline{q}$. The sequence $\left(q_1^{(i)}\right)_{i=0}^{\infty}$ is monotone and bounded. It follows from the Monotone Convergence Theorem that $\lim_{i\to\infty} q_1^{(i)} = \hat{q}_1 \leq \overline{q}$. By Berge's Maximum Theorem, the solution to (7) is continuous in q_1. It thus follows that

$$\lim_{i\to\infty} e_1^i(w) \equiv e_1(w) = \arg\max_e e \cdot \left[u(w,\hat{q}_1) + v(w, r_1(w))\right] - c(e),$$

where $\hat{q}_1 = \lim_{i\to\infty} q_1^i$. By construction, $e_1(w) = e_1^*(w)$, for all $w \in W_1$. Repeating this argument for all other W_k with $q_k^{(0)} = \hat{q}_{k-1}$ yields the result, noting that $q_k^{(1)} \geq \hat{q}_{k-1}$, since $u(w,q)+v(w,r)$ (and hence $e^*(w)$) is strictly increasing in w, and for all $w \in W_k$, $w \geq \overline{w}_{k-1}$.

□

A.2 Proofs (Section 3)

Proof of Lemma 1: **(i):** If I is not a partition, there exist $w_0, w_1 \in [\underline{w}, \overline{w}]$, with $w_0 < w_1$ and $g(w_0) = k \neq g(w_1) = \emptyset$, where g is an assignment function corresponding to I. By Assumptions 1 and 3, $U(w_1, k, \mathscr{F}_k) > U(w_0, k, \mathscr{F}_k) \geq \underline{u}$. A contradiction.

Suppose now the support of some F_k is not convex. As I consists of Borel sets, there exists an open interval $(w_0, w_1) \subset [\underline{w}_k, \overline{w}_k]$, such that for all $w \in (w_0, w_1)$, $g(w) = h \neq k$. Take such a type w. In equilibrium, $U(w,h,\mathscr{F}_h) \geq U(w,k,\mathscr{F}_k)$. Define $\hat{w} \equiv \inf\{x \in (w, \overline{w}] : x > w, g(x) = k\}$. By construction, $\hat{w} > w$. As all types in $[w, \hat{w})$ are not in the support of F_k, $r_k(w) = r_k(\hat{w})$. Suppose $q_h \geq q_k$. Incentive compatibility requires that $p_h \geq p_k$, as $u(\underline{w}_k, q_h) + v(\underline{w}_k, 0) > u(\underline{w}_k, q_k) + v(\underline{w}_k, 0)$ and hence $e_h^*(\underline{w}_k)\cdot\left[u(\underline{w}_k, q_h)+v(\underline{w}_k, 0)\right] - c(e_h^*(\underline{w}_k)) > e_k^*(\underline{w}_k)\cdot\left[u(\underline{w}_k, q_k) + v(\underline{w}_k, 0)\right] - e_k^*(\underline{w}_k)$. But if $p_h \geq p_k$, then $u(w,q_h) + v(w, r_h(w)) - u(w, q_k) - v(w, r_k(w)) = \delta \geq 0$. By Assumption 2, $u(\hat{w}, q_h) + v(\hat{w}, r_h(w)) - u(\hat{w}, q_k) - v(\hat{w}, r_k(w)) \geq \delta \geq 0$ and by Lemma 3:

$$\begin{aligned} &e_h^*(\hat{w})\cdot\left[(u(\hat{w}, q_h) + v(\hat{w}, r_h(w))\right] - c(e_h^*(\hat{w})) - \left(e_h^*(\hat{w})\cdot\left[u(\hat{w}, q_k) + v(\hat{w}, r_k(w))\right] - c(e_h^*(\hat{w}))\right) \\ \geq &e_h^*(w)\cdot\left[(u(w, q_h) + v(w, r_h(w))\right] - c(e_h^*(w)) - \left(e_h^*(w)\cdot\left[u(w, q_k) + v(w, r_k(w))\right] - c(e_h^*(w))\right). \end{aligned}$$

As all types in $[\underline{w}, \overline{w}]$ have strictly positive density, $r_h(\hat{w}) > r_h(w)$. As utility is strictly increasing in r (Assumption 1), type $\hat{w}$ must strictly prefer group h. A contradiction.

Suppose instead $q_k > q_h$. Using the same argument as before, this requires $p_k > p_h$. It follows that $u(\underline{w}_k, q_k) + v(\underline{w}_k, 0) - u(\underline{w}_k, q_h) - v(\underline{w}_k, r_h(\underline{w}_k)) = \delta > 0$. Let $w^* = \inf\{w : w > \underline{w}_k, g(w) = h\}$. By Definition of $\underline{w}_k$, $w^* > \underline{w}_k$. As $(\underline{w}_k, w^*)$ is not in the support of F_h, $r_h(w^*) = r_h(\underline{w}_k)$ while $r_k(w^*) > r_k(\underline{w}_k)$. But then by Assumptions 1 and 2, we have $u(w^*, q_k) + v(w^*, r_k(w^*)) - u(w^*, q_h) - v(w^*, r_h(w^*)) > \delta$. This again leads to a contradiction.

(ii): Suppose not and $\mathscr{F}_h \neq \mathscr{F}_l$ but $q_h = q_l = q$. Following the argument from (i), this implies $p_h = p_l$. As $F_h \neq F_l$, $\underline{w}_l \neq \underline{w}_h$. Suppose WLOG that $\underline{w}_l < \underline{w}_h$. This implies $r_l(w) > 0, \forall w > \underline{w}_l$ but $r_h(w) = 0, \forall w \in [\underline{w}_l, \underline{w}_h]$. As $v(\cdot)$ is strictly increasing in r, $u(\underline{w}_h, q) + v(\underline{w}_h, r_l(\underline{w}_h)) > u(\underline{w}_h, q) + v(\underline{w}_h, 0)$. A contradiction.

(iii): If $q_h > q_l$, then using the argument from (i), $p_h > p_l$. Suppose to the contrary that

$\overline{w}_l > \underline{w}_h$. Again by (i), the support of both groups must be an interval, and so $\underline{w}_l \geq \overline{w}_h$. By Assumption 2, if $u(\overline{w}_h, q_h) + v(\overline{w}_h, 1) - u(\overline{w}_h, q_l) - v(\overline{w}_h, 0) > 0$, then this holds strictly for all $w > \overline{w}_h$, contradicting $\underline{w}_l \geq \overline{w}_h$. It follows that $q_l < q_h \implies \overline{w}_l \leq \underline{w}_h$.

Suppose now $\overline{w}_l \leq \underline{w}_h$. By Assumption 4, $q_h \geq q_l$. As $\mathscr{F}_h \neq \mathscr{F}_l$, it follows from (ii) that $q_h \neq q_l$ and thus $q_h > q_l$. From (i), it follows that $p_h > p_l$.

Finally, suppose $p_h > p_l$. Clearly, $q_h \neq q_l$. Suppose $q_l > q_h$. Then $e_h^*(\underline{w}_h) \cdot \left[u(\underline{w}_h, q_h) + v(\underline{w}_h, 0)\right] - c(e_h^*(\underline{w}_h)) - p_h < e_l^*(\underline{w}_h) \cdot \left[u(\underline{w}_h, q_l) + v(\underline{w}_h, r_l(\underline{w}_h))\right] - c(e_l^*(\underline{w}_h)) - p_l$, implying that $\underline{w}_h$ must prefer $\mathscr{F}_l$. A contradiction. It follows that $p_h > p_l \implies q_h > q_l$, which completes the chain. □

Proof of Lemma 2. Take any interval partition I of $[\underline{w}_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$ with corresponding $g(w)$. First, suppose preferences are given by U_q. Using (3), membership prices can be written as:

$$\begin{aligned} p_k &= e_k(\underline{w}_k) \cdot u(\underline{w}_k, q_k) - c(e_k(\underline{w}_k)) \\ &\quad - \left(e_{k-1}(\underline{w}_k) \cdot u(\underline{w}_k, q_{k-1}) - c(e_{k-1}(\underline{w}_k)) - p_{k-1}\right), \quad \forall k \in A \setminus \{\emptyset\}, k > 1 \\ p_1 &= e_1(\underline{w}_1) \cdot u(\underline{w}_1, q_1) - c(e(\underline{w}_1)) - \underline{u}. \end{aligned} \tag{8}$$

Sufficiency: IC requires that in any equilibrium $q_1 \leq q_2 \leq ....$ Furthermore, $u(\hat{w}, q) > u(w, q)$, if $\hat{w} > w$ (Assumption 1). Consequently, for any $\hat{w} \geq \underline{w}_1 > w$,

$$e_1(\hat{w}) \cdot u(\hat{w}, q_1) - c(e_1(\hat{w})) - p_1 \geq \underline{u} > e_1(w) \cdot u(w, q_1) - c(e_1(w)) - p_1.$$

All types lower than $\underline{w}_1$ prefer $\mathscr{F}_\emptyset$ to $\mathscr{F}_1$, while for all higher types, the opposite is true. Now consider any social group $\mathscr{F}_k$ with $k > 1$. Using (8), Assumption 1 (complementarity) and Lemma 3, for all $\hat{w} \geq \underline{w}_k > w$, the following holds:

$$U(\hat{w}, k, \mathscr{F}_k) - U(\hat{w}, k-1, \mathscr{F}_{k-1}) \geq 0 > U(w, k, \mathscr{F}_k) - U(w, k-1, \mathscr{F}_{k-1}).$$

As $e_k(w) = e_k^*(w)$ for all $w \in [\underline{w}_1, \overline{w}]$, we can conclude that all types $w > \underline{w}_k$ prefer k to $k-1$ and vice versa. Combining this with the result on $\mathscr{F}_1$, we can conclude that IC is satisfied and $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium.

Necessity: Suppose $(I, \mathbf{e}, \mathbf{p})$ is an equilibrium. Clearly, $e_1^*(w) = e_1(w)$ for all $w \in [\underline{w}_1, \overline{w}]$ is necessary for equilibrium. Note further that if $w_1 = \underline{w}$, then $\mathscr{F}_1$ is preferred to $\mathscr{F}_\emptyset$ as long as $p_1 \leq e_1(\underline{w}) \cdot u(\underline{w}, q_1) - c_1(e_1(\underline{w})) - \underline{u}$. Now suppose there are at least two adjacent social groups $\mathscr{F}_h$ and $\mathscr{F}_l$, each with interval support. Assume WLOG that $\overline{w}_l = \underline{w}_h$. It follows directly from the equilibrium definition and continuity that type $\underline{w}_h$ must be indifferent between both groups. This requires $e_h(\underline{w}_h) \cdot u(\underline{w}_h, q_h) - c(e_h(\underline{w})) - p_h = e_l(\underline{w}_h) \cdot u(\underline{w}_h, q_l) - c(e_l(\underline{w}_h)) - p_l$, which can be rearranged to $p_h = e_h(\underline{w}_h) \cdot u(\underline{w}_h, q_h) - c(e_h(\underline{w})) - \left(e_l(\underline{w}_h) \cdot u(\underline{w}_h, q_l) - c(e_l(\underline{w}_h))\right) + p_l$. Finally, in case $\underline{w}_1 > \underline{w}$, the same argument applies for $\mathscr{F}_\emptyset$ and $\mathscr{F}_1$ (with utility of choosing $\mathscr{F}_\emptyset$ equal $\underline{u}$). Prices thus must be as defined in (3).

Now instead consider preferences U:

Necessity: This follows from the same argument as above using

$$e_h(\underline{w}_h)\cdot\left[u(\underline{w}_h,q_h)+v(\underline{w}_h,0)\right]-c(e_h(\underline{w}_h))-p_h = e_l(\underline{w}_h)\cdot\left[u(\underline{w}_h,q_l)+v(\underline{w}_h,1)\right]-c(e_l(\underline{w}_h))-p_l$$

as the relevant condition for each cut-off type. To show that sufficiency fails, consider two groups $\mathscr{F}_l$, $\mathscr{F}_h$, with $q_l = q_h = q$, and support over $[\underline{w}_l, w]$ and $[w, \overline{w}_h]$. As type w must be indifferent, and noting that $v(w,1) > v(w,0)$ and hence $e_h(w) \le e_l(w)$, we have:

$$\begin{aligned}p_h &= e_h(w)\cdot\left[u(w,q)+v(w,0)\right]-c(e_h(w))-e_l(w)\cdot\left[u(w,q)+v(w,1)\right]+c(e_l(w))+p_l\\ &\le e_h(w)\cdot\left[v(w,0)-v(w,1)\right]+p_l < p_l.\end{aligned}$$

By Lemma 1, this cannot be an equilibrium. □

Proof of Proposition 1. Equilibrium existence follows directly from Lemma 2 and Lemma 4, noting that for preferences with status concern, a price satisfying incentive compatibility (IC) can always be found if there is only a single group.

I show the existence of an upper bound using U_α. Taking α to 0 and 1 gives the results for U_q and U_r respectively. It follows from Lemma 1 that in any equilibrium, the supports of the social groups form an interval partition of $[\underline{w}_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$. It is a necessary condition for IC, that for any two adjacent social groups $\mathscr{F}_h, \mathscr{F}_l$ with $q_h > q_l$, we have

$$\begin{aligned}&e_h(\underline{w}_h)\cdot\left[(1-\alpha)\cdot u(\underline{w}_h,q_h)+\alpha\cdot v(\underline{w}_h,0)\right]-c(e_h(\underline{w}_h))\\ >\ &e_l(\underline{w}_l)\cdot\left[(1-\alpha)\cdot u(\underline{w}_h,q_l)+\alpha\cdot v(\underline{w}_h,1)\right]-c(e_l(\underline{w}_h)),\end{aligned}$$

where $\underline{w}_h = \overline{w}_l$. Note that the inequality must be strict as otherwise the difference in prices must be 0 (Lemma 2). But then type $\underline{w}_l$ with $r_l(\underline{w}_l) = r_h(\underline{w}_l) = 0$ strictly prefers $\mathscr{F}_h$, since $q_h > q_l$. It follows that:

$$(1-\alpha)\cdot u(\underline{w}_h,q_h)+\alpha\cdot v(\underline{w}_h,0) > (1-\alpha)\cdot u(\underline{w}_h,q_l)+\alpha\cdot v(\underline{w}_h,1), \tag{9}$$

For a given $\alpha \in (0,1]$, $q \in [\underline{q}, \overline{q})$, and $w \in [\underline{w}, \overline{w}]$, we can define the set of possible quality differences, such that (9) is satisfied. In particular, let $\Delta_\alpha(q,w)$ be the set of all $\delta \in \mathbb{R}$, for which the following holds: $(1-\alpha)\cdot u(w,q+\delta)+\alpha\cdot v(w,0) > (1-\alpha)\cdot u(w,q)+\alpha\cdot v(w,1)$. Note that this set is empty if

$$\lim_{\delta\to\infty}(1-\alpha)\cdot u(w,q+\delta)+\alpha\cdot v(w,0) < (1-\alpha)\cdot u(w,q)+\alpha\cdot v(w,1). \tag{10}$$

In this case, for notational convenience and without loss, assume $\Delta_\alpha(q,w) = \overline{q}-\underline{q}$ (or any larger value). As $v(w,1) > v(w,0)$, any element of this set is strictly greater than 0 for all $w \in [\underline{w}, \overline{w}]$ and q. As it is also non-empty by construction, it has a (non-zero) lower bound. Denote its greatest lower bound by $\underline{\Delta}_\alpha(q,w) \equiv \inf \Delta_\alpha(w,q)$. If there are two adjacent social groups with cut-off type w and group qualities q and $q+\delta$, then if $\delta \le \underline{\Delta}_\alpha(w,q)$, IC fails.

For a given w, we can define $\underline{\Delta}_\alpha(w) \equiv \inf\{\underline{\Delta}_\alpha(w,q) : q \in [\underline{q},\overline{q}]\}$; the smallest such difference for any possible q. Since for every $q \in [\underline{q},\overline{q}]$, it is the case that $\underline{\Delta}_\alpha(w,q) > 0$, this lower bound is also strictly greater than 0. As a final step, we can define the equivalent lower bound across all types as $\underline{\Delta}_\alpha \equiv \inf\{\underline{\Delta}_\alpha(w) : w \in [\underline{w},\overline{w}]\}$. Following the same argument as before, this is also strictly positive.

As a final step, it is shown that in equilibrium, the number of social groups is bounded above by $\overline{N}_\alpha \equiv \left\lceil \frac{\overline{q}-\underline{q}}{\underline{\Delta}_\alpha} \right\rceil$, with $\lceil x \rceil$ denoting the ceiling function: Suppose not and there exists an equilibrium group structure with $N > \overline{N}_\alpha$ social groups. Then as

$$N > \overline{N}_\alpha \geq \frac{\overline{q}-\underline{q}}{\underline{\Delta}_\alpha},$$

there must be a pair of adjacent social groups with $q_h > q_l$, and $q_h - q_l < \underline{\Delta}_\alpha$. By definition, $q_h - q_l < \underline{\Delta}_\alpha(\underline{w}_h) \leq \underline{\Delta}_\alpha(\underline{w}_h, q_l)$, which implies that IC is violated.

To see that for $\alpha = 1$ the upper bound is 1, note that $\lim_{\alpha\to 1} \Delta_{\alpha(q,w)} = \overline{q}-\underline{q}$ for all $w \in [\underline{w},\overline{w}]$, following the convention established for (10). It follows that $\overline{N}_1 = 1$.

Finally, to show that no upper-bound exists for U_q, take any interval partition I of $[\underline{w}_1,\overline{w}]$, with $\underline{w}_1 \geq \underline{w}$. It follows from Lemma 2 and Lemma 4, that we can find prices $\mathbf{p}$ and engagement levels $\mathbf{e}$, such that the corresponding $(I,\mathbf{e},\mathbf{p})$ is an equilibrium. Since I is an arbitrary interval partition, it can be arbitrarily fine. □

Proof of Corollary 1.1. It follows from linearity in α and $v(w,0) < v(w,1)$, $\forall w \in [\underline{w},\overline{w}]$, that there exists $\alpha' < 1$, such that for all $w \in [\underline{w},\overline{w}]$ and $q \in [\underline{q},\overline{q}]$, we have:

$$(1-\alpha')\cdot u(w,q+\overline{q}-\underline{q}) + \alpha'\cdot v(w,0) < (1-\alpha')\cdot u(w,q) + \alpha'\cdot v(w,1).$$

By definition, $\underline{\Delta}_{\alpha'} = \overline{q}-\underline{q}$, and thus $\overline{N}_{\alpha'} = 1$. It follows that $\overline{N}_\alpha = 1$ for all $\alpha > \alpha'$. □

Proof of Corollary 1.2. This follows directly from the last argument of Proposition 1. □

A.3 Proofs (Section 4)

Proof of Proposition 2. Take two groups $\mathscr{F}_h$ and $\mathscr{F}_l$ with $\overline{w}_l = \underline{w}_h = \tilde{w}$ from a group structure with $|I| > 1$. Let $\alpha = 1$. We can compare the welfare from the two groups to the welfare from an integrated group $\mathscr{F}_k$ with support over $[\underline{w}_l,\overline{w}_h]$. A more detailed decription of this construction can be found in the proof of Lemma OA2 (Online Appendix). Consider the following constructions for types $w > \tilde{w}$:

$$\Delta_+^k \equiv \int_{\tilde{w}}^{\overline{w}_h} e_k^*(w)\cdot v\big(w,r_k(w)\big) - c(e_k^*(w)) - \Big(\tilde{e}(r_k(w))\cdot v\big(\tilde{w},r_k(w)\big) - c(\tilde{e}(r_k(w)))\Big)dF(w),$$

$$\Delta_+^h \equiv \int_{w^*}^{\overline{w}_h} e_h^*(w)\cdot v\big(w,r_h(w)\big) - c(e_h^*(w)) - \Big(\tilde{e}(r_h(w))\cdot v\big(\tilde{w},r_h(w)\big) - c(\tilde{e}(r_h(w)))\Big)dF(w),$$

where $\tilde{e}(r)$ describes the optimal engagement of type $\tilde{w}$ given rank r. From complementarity (Assumption 1), Lemma 3, and $r_k(w) \geq r_h(w)$, we can conclude that $\Delta_+^k \geq \Delta_+^h$, with the inequality strict for strict complementarity between type and status. The equivalent construction for types $w < \tilde{w}$ is:

$$\Delta_-^k \equiv \int_{w_l}^{\tilde{w}} \tilde{e}(r_k(w)) \cdot v\big(\tilde{w}, r_k(w)\big) - c(\tilde{e}(r_k(w))) - \Big(e_k^*(w) \cdot v\big(w, r_k(w)\big) - c(e_k^*(w))\Big) dF(w),$$

$$\Delta_-^l \equiv \int_{w_l}^{\tilde{w}} \tilde{e}(r_l(w)) \cdot v\big(\tilde{w}, r_l(w)\big) - c(\tilde{e}(r_l(w))) - \Big(e_l^*(w) \cdot v\big(w, r_l(w)\big) - c(e_l^*(w))\Big) dF(w),$$

Using the same argument as before and noting that $r_l(w) \geq r_k(w)$, we can conclude that $\Delta_-^l \geq \Delta_-^k$. It follows

$$\begin{aligned}
\int_{\underline{w}_l}^{\overline{w}_h} e_k^*(w) \cdot v(w, r_k(w)) - c(e_k^*(w)) dF(w) &= \kappa \int_0^1 \tilde{e}(r) \cdot v(\tilde{w}, r) - c(\tilde{e}(r)) dr + \Delta_+^k - \Delta_-^k \\
&\geq \kappa \int_0^1 \tilde{e}(r) \cdot v(\tilde{w}, r) - c(\tilde{e}(r)) dr + \Delta_+^h - \Delta_-^l \\
&= \int_{\underline{w}_l}^{\overline{w}_l} e_l^*(w) \cdot v(w, r_l(w)) - c(e_l^*(w))) dF(w) \\
&\quad + \int_{\underline{w}_h}^{\overline{w}_h} e_h^*(w) \cdot v(w, r_h(w)) - c(e_h^*(w))) dF(w),
\end{aligned}$$

with $\kappa = F(\overline{w}_h) - F(\underline{w}_l)$. The inequality is strict for strict complementarity between type and status. Since U_α is continuous in α, and for any $w \in [\underline{w}_l, \overline{w}_h]$, $u(w, q_h) - u(w, q_k)$ is bounded by the (finite) difference $u(\overline{w}, \overline{q}) - u(\overline{w}, \underline{q})$, this also holds for large enough $\alpha < 1$. There then exists an $\underline{\alpha}$, such that for all $\alpha > \underline{\alpha}$, the unconstrained welfare-maximising partition consists of a single group. The argument extends to the welfare-maximising equilibrium provision, noting that any singleton $I = \{W_1\}$ can be achieved in equilibrium if $W_1 = [w_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$.

Profit maximisation: Suppose $(I, \mathbf{e}, \mathbf{p})$ is such that $|I| > 1$. Let $\mathscr{F}_h$ and $\mathscr{F}_l$ be two groups with $\overline{w}_l = \underline{w}_h = \tilde{w}$. It follows from Lemma 2 and profit maximisation that:

$$\begin{aligned}
p_h(\alpha) &= p_l(\alpha) \quad e_h^*(\tilde{w}) \cdot \big[(1-\alpha) u(\tilde{w}, q_h) + \alpha v(\tilde{w}, 0)\big] - c(e_h^*(\tilde{w})) \\
&\qquad - \Big(e_l^*(\tilde{w}) \cdot \big[(1-\alpha) u(\tilde{w}, q_l) + \alpha v(\tilde{w}, 1)\big] - c(e_l^*(\tilde{w}))\Big), \\
p_l(\alpha) &= e_l^*(\underline{w}_l) \cdot \big[(1-\alpha) u(\underline{w}_l, q_l) + \alpha v(\underline{w}_l, 0)\big] - c(e_l^*(\underline{w}_l)) - \underline{u}.
\end{aligned}$$

Let $\mathscr{F}_k$ be a group whose support is the union of the supports of $\mathscr{F}_l$ and $\mathscr{F}_h$. The profit maximising membership price for this group is $p_k(\alpha) = e_k^*(\underline{w}_l) \cdot \big[(1-\alpha) u(\underline{w}_l, q_k) + \alpha v(\underline{w}_l, 0)\big] - c(e_k^*(\underline{w}_l)) - \underline{u}$. The maximum benefit from offering a more segregated group structure is achieved if $q_h = \overline{q}$, while $q_k = q_l = \underline{q}$. But since $v(\tilde{w}, 1) > v(\tilde{w}, 0)$, $\lim_{\alpha \to 1} p_h(\alpha) - p_l(\alpha) < 0$, and $\lim_{\alpha \to 0} p_l(\alpha) - p_k(\alpha) = 0$, the revenue difference between offering groups $\mathscr{F}_h$ and $\mathscr{F}_l$, and the integrated $\mathscr{F}_k$, is strictly negative. It follows that integrating the lowest two groups of any group structure strictly increases profits. This implies that the profit maximising partition contains only one group. As before, it follows from continuity of U_α in α that this holds for large enough $\alpha < 1$, which implies the existence of some $\underline{\alpha}$. As before, such an equilibrium exists. □

Proof of Corollary 2.1. This follows immediately from the argument of Proposition 2, replacing the weight α with λ. □

Proof of Proposition 3. Given U_α, suppose that $\lambda = \alpha$. Let $(I, \mathbf{e}, \mathbf{p})$ be the corresponding welfare-maximising equilibrium group provision. Take any $\alpha' > \alpha$. Let $(I', \mathbf{e}', \mathbf{p}')$ be the welfare-maximising equilibrium provision for $U_{\alpha'}$ given λ. Furthermore, let $(I', \hat{\mathbf{e}}, \hat{\mathbf{p}})$ be the welfare-maximising equilibrium fixing I', but for preferences U_α.

It is now shown that if I' is an equilibrium partition for $U_{\alpha'}$ and quality is independent of engagement, then it must be an equilibrium partition for any $\alpha < \alpha'$, meaning that an equilibrium provision $(I', \hat{\mathbf{e}}, \hat{\mathbf{p}})$ indeed exists: First note that I' fully determines q_i for each social group $\mathscr{F}_i$. IC requires that given some $\underline{w}_j$, the following holds for all $w < \underline{w}_j$ with $g(w) = i \neq \emptyset$:

$$\begin{aligned} p_j - p_i \geq f_w(\alpha) \equiv \quad & e_j^*(w)\big[(1-\alpha)u(w, q_j) + \alpha v(w, 0)\big] - c(e_j^*(w)) \\ & - \Big(e_i^*(w)\big[(1-\alpha)u(w, q_i) + \alpha v(w, r_i(w))\big] - c(e_i^*(w))\Big), \end{aligned}$$

where according to Lemma 2,

$$\begin{aligned} p_j - p_i = f_{\underline{w}_j}(\alpha) \equiv \quad & e_j^*(\underline{w}_j)\big[(1-\alpha)u(\underline{w}_j, q_j) + \alpha v(\underline{w}_j, 0)\big] - c(e_j^*(\underline{w}_j)) \\ & - \Big(e_i^*(\underline{w}_j)\big[(1-\alpha)u(\underline{w}_j, q_i) + \alpha v(\underline{w}_j, 1)\big] - c(e_i^*(\underline{w}_j))\Big). \end{aligned}$$

It follows from strict complementarity between q and w that $f_w(0) < f_{\underline{w}_j}(0)$. Moreover, since $r_i(w) \leq 1 = r_i(\underline{w}_j)$, it follows from (weak) complementarity between w and r that $f_w(1) \geq f_{\underline{w}_j}(1)$. As $c(e)$ is quadratic, f_w and $f_{\underline{w}_j}$ are second-order polynomials. Accordingly, there is at most one $\alpha' \in (0,1)$ such that $f_w(\hat{\alpha}) = f_{\underline{w}_j}(\hat{\alpha})$. Consequently, if $f_{\underline{w}_j}(\alpha') \geq f_w(\alpha')$, then this also holds for all $\alpha \in [0, \alpha')$. This means $(I, \mathbf{e}, \mathbf{p})$ achieves (weakly) higher welfare than $(I', \hat{\mathbf{e}}, \hat{\mathbf{p}})$. Since q is independent of e, $\hat{\mathbf{e}}$ is the unconstrained maximum given I' and, as $\alpha' \neq \lambda$, $\hat{\mathbf{e}} \neq \mathbf{e}'$. We can thus conclude that $(I, \mathbf{e}, \mathbf{p})$ achieves strictly higher welfare than $(I', \mathbf{e}', \mathbf{p}')$. □

Proof of Proposition 4. To prove the first claim, we construct a suitable quality function q and distribution F. Take any $[\underline{w}, \overline{w}]$. For any (Borel)–set $S \subseteq [\underline{w}, \overline{w}]$, let $I_S = [\underline{w}_S, \overline{w}_S]$ be the smallest interval such that the measure of $S \cap I_S$ equals the measure of S. I_S contains all types in S with probability one, but single deviations do not affect quality. Define q as follows:

$$q(S) = \begin{cases} q_l & \text{if } \underline{w}_S < w^* \\ q_h & \text{if } \underline{w}_S \geq w^* \end{cases}$$

for some $q_h > q_l$ and $w^* \in (\underline{w}, \overline{w})$. It follows from Lemma 1 that there can be at most two social groups in equilibrium. Given this q, it is never optimal for a planner to offer a group with the lowest type $\underline{w}_1 \in (\underline{w}, w^*)$ since extending such a group to include all lower types increases welfare. Similarly, excluding agents with types higher than w^* is equally suboptimal. Consider the partitions $I = \{[w^*, \overline{w}]\}$ and $I' = \{[\underline{w}, \overline{w}]\}$. The difference in aggregate utility can be written as:

$$\begin{aligned}\Delta_s(q_h, F) = \quad & \int_{w^*}^{\overline{w}} e_h(w) \cdot \left[u(w, q_h) + v(w, r_h(w))\right] - c(e_h(w)) dF(w) \\ & - \int_{w^*}^{\overline{w}} e_l(w) \cdot \left[u(w, q_l) + v(w, r_l(w))\right] - c(e_l(w)) dF(w) \\ & - \int_{\underline{w}}^{w^*} e_l(w) \cdot \left[u(w, q_l) + v(w, r_l(w))\right] - c(e_l(w)) - \underline{u}\, dF(w).\end{aligned} \tag{11}$$

Fixing some q_l, we can choose $\overline{q}_h > q_l$ such that $\max_{w \in [w^*, \overline{w}]} u(w, \overline{q}_h) + v(w, 0) - u(w, q_l) - v(w, 1) \leq 0$. As $v(w, 0) - v(w, 1) < 0$, such a $\overline{q}_h$ exists. For all $q_h < \overline{q}_h$, there can be at most one group. The welfare-maximising partition is then I or I'. Let F_1 be a distribution with strictly positive support over $[\underline{w}, \overline{w}]$. We can construct the following sequence of distributions $\{F_n\}_{n=1}^{\infty}$:

$$\frac{F_{n+1}(w)}{F_n(w)} = \frac{1}{k}, \quad \forall w \leq w^*.$$

where $k > 1$. Furthermore,

$$\frac{F_{n+1}(w) - F_{n+1}(w^*)}{F_n(w) - F_n(w^*)} = \left(1 - \frac{1}{k}\right) \cdot F_n(w^*) \cdot \left[1 - F_n(w^*)\right]^{-1}, \quad \forall w > w^*$$

Notice that $\lim_{n \to \infty} F_n(w^*) = 0$ and hence at the limit, $r_h(w) = r_l(w)$. However, as utility is strictly increasing in q, for any $q_h > q_l$, $e_h(w) \geq e_l(w)$ and hence $\lim_{n \to \infty} \Delta_s(q_h, F_n) > 0$. It follows from continuity that for any $q_h \in (q_l, \overline{q}_h)$, there exists N such that for $n > N$, $\Delta_s(q_h, F_n) > 0$. For such a distribution and q, social exclusion is a constrained welfare maximum.

Next, it is shown that social exclusion cannot be optimal for U_q: let I be an interval partition of $[\underline{w}_1, \overline{w}]$ and $(I, \mathbf{e}, \mathbf{p})$ the corresponding equilibrium. Let $I' = I \cup \{[\underline{w}, \underline{w}_1]\}$, meaning I' extends I with an element that includes all types not part of I. Let $\mathbf{e}'$ be such that $e'_k(w) = e_k(w)$ for all intervals in I. It follows from Lemma 2 and Lemma 4 that we can find an IC $e'_0(w)$ for all $w \in [\underline{w}, \underline{w}_1]$ and $\mathbf{p}' \geq 0$ such that $(I', \mathbf{e}', \mathbf{p}')$ is an equilibrium provision. By Assumption 3, utility of membership exceeds $\underline{u}$ for (almost) every $w \in [\underline{w}, \underline{w}_1]$. As utility net of payments remains unchanged for all other types, welfare is higher under I'.

Finally, it is shown that for U_r, welfare is maximised with a single group that includes all types: it follows from Proposition 1 that for U_r, there can be at most one group. Suppose $I = \{[\underline{w}_1, \overline{w}]\}$, with $\underline{w}_1 > \underline{w}$. Denote the corresponding social group by $\mathscr{F}_1$. Compare this to $I' = \{[\underline{w}_0, \overline{w}]\}$ with social group $\mathscr{F}'_1$. Clearly, for all $w \in [\underline{w}_1, \overline{w})$, $r'_1(w) > r_1(w)$, which increases utility. Furthermore, for all $w' \notin [\underline{w}_1, \overline{w})$, $\hat{U}(w', 1', \mathscr{F}_{1'}) \geq \underline{u}$. Welfare is thus maximised for $I^* = \{[\underline{w}, \overline{w}]\}$. For $p_1^* = 0$, the corresponding $(I^*, \mathbf{e}^*, \mathbf{p})$ is an equilibrium provision. □

Proof of Corollary 4.1. This is shown as part of the proof of Proposition 4. □

Proof of Corollary 4.2. To show this, we can extend the construction in the proof of Proposition 4. This implicitly assumes that $\lambda = \alpha \in (0,1)$. Starting from an arbitrary u, v, q_l, and $q_h < \overline{q}_h$ (see proof of Proposition 4), define the following preferences for any $\tau \leq 1$:

$$u_\tau(w,q) = \begin{cases} u(w^*,q_h) - \tau \cdot \big(u(w^*,q_h) - u(w,q)\big), & \text{if } w \leq w^* \\ u_k(w^*,q) + \int_{w^*}^{w} \frac{\partial}{\partial w} u(w,q)\,dw, & \text{otherwise.} \end{cases}$$

Note that $u_1(w,q) = u(w,q)$. Utility u_τ rescales the slope of the utility function for all types $w \leq w^*$, keeping $u(w^*,q_h)$ fixed. By construction, for every positive $\delta \leq u(w^*,q_h) - u(w,q_l)$, we can find τ such that $u_\tau(w^*,q_h) - u_\tau(\underline{w},q_l) = \delta$. For simplicity, we denote these preferences by u_δ. Furthermore, suppose status concern is independent of type, meaning for all $w, w' \in [\underline{w}, \overline{w}]$ and any $r \in [0,1]$, we have $v(w,r) = v(w',r)$. It follows from Lemma 2 that if prices are profit maximising, the price difference between the corresponding social groups of $I = \{[\underline{w}_1, \overline{w}]\}$ and $I' = \{[\underline{w}, \overline{w}]\}$ equals:

$$\begin{aligned} \Delta_p(\delta) \equiv \quad & e_1(w^*) \cdot \big[u_\delta(q_h, w^*) + v(w^*,0)\big] - c\big(e_1(w^*)\big) \\ & -\Big(e_1'(w^*) \cdot \big[u_\delta(q_l, w^*) + v(w^*,0)\big] - c\big(e_1'(w^*)\big)\Big) \end{aligned}$$

The difference in revenue can be expressed as:

$$\Delta_m(\delta,F) = \big(1 - F(w^*)\big) \cdot \Delta_p(\delta) - F(w^*) \cdot \big(u_\delta(\underline{w},q_l) + v(\underline{w},0) - \underline{u}\big).$$

The difference in utility is as in (11). Using the assumptions here, we can establish that:

$$\begin{aligned} \Delta_s(\delta,F) > \quad & \int_{w^*}^{\overline{w}} e_1(w) \cdot \big[u(w,q_h) + v(w,r(w))\big] - c(e_1(w))\,dF(w) \\ & - \int_{w^*}^{\overline{w}} \hat{e}_1'(w) \cdot \big[u(w^*,q_h) + u(w,q_l) - u(w^*,q_l) + v(w,r'(w))\big] - c(\hat{e}_1'(w))\,dF(w) \\ & - F(w^*) \cdot \Big(e_1'(w^*) \cdot \big[u(w^*,q_h) + v(w^*,F(w^*))\big] - c(e_1'(w^*)) - \underline{u}\Big), \end{aligned}$$

making use of the fact that, by construction, $u_\delta(w,q) - u_\delta(w',q) = u(w,q) - u(w',q)$ for all $w, w' \geq w^*$, and $\hat{e}_1'(w)$ being defined as the optimal engagement choice for utility level $u(w^*,q_h) + u(w,q_l) - u(w^*,q_l) + v(w,r'(w))$. Denote the right-hand side by $\underline{\Delta}_s(F)$, which does not depend on δ.

Observe further that if $\frac{F(w)-F(w^*)}{w-w^*}$ is sufficiently large for all $w > w^*$, then excluding types higher than w^* is not profit maximising (and trivially suboptimal for a planner). Starting from such an F_1, we can construct a sequence of distributions $\{F_n\}_{n=1}^{\infty}$ in the same way as in the proof Proposition 4. Note that the sequence is such that if excluding types above w^* is not profit maximising for some $\underline{n}$, then this holds for all $n > \underline{n}$. The profit maximising partition for any such F_n is thus either I or I'. By the argument from the proof of Proposition 4, there exists n^* such that for all $n > n^*$, $\underline{\Delta}_S(F_n) > 0$. For any n, however, there exists δ_n such that for all $\delta < \delta_n$, $\Delta_m(\delta, F_n) < 0$. We can thus find $\delta > 0$ and $n \geq 1$ such that $\Delta_s(\delta, F_n) > \underline{\Delta}_S(F_n) > 0 > \Delta_m(\delta, F_n)$. □

Proof of Proposition 5. **Part (i)**: If $|I| = 1$, there is a single social group ($\mathscr{F}_1$). As $(I, \mathbf{e}, \mathbf{p})$ is a competitive group structure, there is a firm $z \in Z$ with $\zeta(1) = z$. If $p_1 > 0$ then offering any $p'_1 \in (0, p_1)$ satisfies IC and is a Pareto improvement and strictly beneficial to any firm $z' \neq z$. For $p_1 = 0$, no such Pareto improvement exists that would also satisfy the 0-profit condition.

Suppose now $|I| = n > 1$. Lemma 2 and the 0-profit condition imply that $p_n > 0$. Suppose now $p_1 \geq 0$. Then there exists a price vector $\mathbf{p}'$ with $p'_k = p_k - \epsilon$ for all $k \in \{1, ..., n\}$ such that $(I, \mathbf{e}, \mathbf{p}')$ is an equilibrium group structure and Pareto-dominates $(I, \mathbf{e}, \mathbf{p})$. Furthermore, for sufficiently small ϵ, this generates positive profits. It follows that $p_1 < 0$ in a competitive equilibrium group structure. As such a symmetric price reduction across all groups is possible as long as profits are positive, the result follows.

Part (ii): Suppose $(I, \mathbf{e}, \mathbf{p})$ is a competitive equilibrium that involves social exclusion. It follows from Lemma 2 that there is a unique and strictly positive $\mathbf{p}$. Providing any social group $\mathscr{F}_k$ thus generates profits $p_k \cdot \left[F(\overline{w}_k) - F(\underline{w}_k)\right] > 0$.

Suppose now there exists an equilibrium $(I', \mathbf{e}', \mathbf{p}')$ that includes an additional group, i.e., $I' = I \cup [w_0, \underline{w}_1]$. Denote the additional group by $\mathscr{F}_0$. As shown in (iii), we can restrict attention to the highest IC engagement levels. Thus $e_k(w) = e'_k(w)$, for all $w \geq \underline{w}_1$ and $k \geq 1$. Since $\hat{U}(w_0, 0, \mathscr{F}_0) \geq \underline{u}$, if there exist $\mathbf{p}'$ that make $(I', \mathbf{e}, \mathbf{p}')$ IC, then such prices exist with $p'_0 \geq 0$. Furthermore, as $\mathscr{F}_1$ is the same for both provisions and $\hat{U}(\underline{w}_1, 0, \mathscr{F}_0) > \hat{U}(w_0, 0, \mathscr{F}_0)$, it follows from Lemma 2 that $p'_k < p_k$, for all $k \geq 1$. $(I', \mathbf{e}', \mathbf{p}')$ is a Pareto-improvement, contradicting that $(I, \mathbf{e}, \mathbf{p})$ is a competitive equilibrium.

Part (iii): Suppose such a distinct group structure exists. Monotonic quality ensures $q'_k \geq q_k$ for all $k \in A$. Since the group structures are distinct, there is at least one $k \in A$ with $q'_k > q_k$. Assume WLOG that $\mathscr{F}_k$ is the lowest quality group with distinct quality, meaning all $l \in A$, with $q_l < q_k$, have $q'_l = q_l$ (if any). From Lemma 2, it follows that $p_l = p'_l$ and $U(\underline{w}_k, k, \mathscr{F}'_k) = U(\underline{w}_k, k, \mathscr{F}_k)$. As utility is strictly increasing in q, it follows that $U(w, k, \mathscr{F}'_k) > U(w, k, \mathscr{F}_k)$ for all $w \in (\underline{w}_k, \overline{w}_k]$, and further (again from Lemma 2), that $U(w, g(w), \mathscr{F}'_{g(w)}) > U(g(w), \mathscr{F}_{g(w)})$, for all $w > \overline{w}_k$. Repeating this argument for any other $\mathscr{F}'_h \neq \mathscr{F}_h$ allows us to conclude that $(I, \mathbf{e}', \mathbf{p}')$ is a Pareto-improvement. $(I, \mathbf{e}, \mathbf{p})$ cannot be a competitive equilibrium. □

A.4 Additional Results (Section 5)

Lemma 5. *For any I, there are type-dependent prices $\mathbf{p}^w$ and engagement choices $\mathbf{e}$, such that the corresponding group structure $(I, \mathbf{e}, \mathbf{p}^w)$ is an equilibrium group provision.*

Proof. Take any I and $\mathbf{e}$, such that $e_k(w) = e^*_k(w)$, for all $k \in A \setminus \{\emptyset\}$ and corresponding w. Existence of such equilibrium $\mathbf{e}$ for any I follows from Lemma 4. Let $p_k(w) = 0$, if $g(w) = k$, with $g(w)$ the assignment function corresponding to I, and $p_k(w) = c \geq \overline{e} \cdot \left[u(\overline{w}, \overline{q}) + v(\overline{w}, 1)\right] - c(\overline{e}) - \underline{u}$ otherwise, where $\overline{e}$ denotes the optimal engagement for type $\overline{w}$ in a group with $\overline{q}$ and $r = 1$. For almost all $w \in [\underline{w}, \overline{w}]$ and $k \in A$, utility is such that $U(w, g(w), \mathscr{F}_{g(w)}) \geq \underline{u} \geq U(w, k, \mathscr{F}_k)$. $(I, \mathbf{e}, \mathbf{p}^w)$ is an equilibrium. □

To generalise the insight from Example 3.1, the following condition is introduced:

Condition C1. *An interval partition I satisfies C1 if there exist $e_k(w) = e_k^*(w)$, for all $k \in A \setminus \{\emptyset\}$ and $w \in [\underline{w}_1, \overline{w}]$, and corresponding $\{\mathscr{F}_k\}$, such that for any two $\mathscr{F}_l, \mathscr{F}_h \in \{\mathscr{F}_k\}$ with $\overline{w}_l \leq \underline{w}_h$:*

$$\int \hat{U}(w,h,\mathscr{F}_h) - \hat{U}(w,l,\mathscr{F}_l)\, dF_h(w) \geq -\Big(\hat{U}(\overline{w}_l, l, \mathscr{F}_l) - \underline{u}\Big).$$

If C1 is satisfied, then - ignoring prices - the average benefit from deviating to a lower quality group is no larger than the benefit from group membership for the highest type in that lower quality group. Ultimately, this is a condition on quality differences. If segregating individuals can (for at least some incentive compatible $\mathbf{e}$) achieve a sufficiently high quality difference between groups, then C1 holds. Proposition 8 shows that in these cases, we can find rank-dependent prices that make the corresponding group structure IC and achieve budget balance - but only if this involves no more than two groups.

Proposition 8. *For any interval partition I of some $[w_1, \overline{w}] \subseteq [\underline{w}, \overline{w}]$, if I satisfies C1 and $|I| \leq 2$, then there exist $\mathbf{p}^r$ and $\mathbf{e}$, such that $(I, \mathbf{e}, \mathbf{p}^r)$ is an equilibrium group provision.*

Proof of Proposition 8. Suppose $|I| = 1$. Since $u(w, q_1) + v(w, 0)$ is increasing in w (Assumption 1), IC is satisfied if $p_1(r) = p_1 = e_1(\underline{w}_1)\big[u(\underline{w}_1, q_1) + v(\underline{w}_1, 0)\big] - c(e_1(\underline{w}_1)) - \underline{u}$, and $e_1(w) = e_1^*(w)$, for all $w \in \mathscr{F}_1$. Suppose instead $|I| = 2$ and let $e_k^*(w)$, for all $k \in A \setminus \{\emptyset\}$, be engagement choices such that the inequality in C1 holds. This requires $q_1 < q_2$, as is necessary in equilibrium. Let $w_k(r) \equiv r_k^{-1}(r)$, the type of rank r (in $\mathscr{F}_k$), if all agents choose according to I. Set $p_1(r) = e_1(w_1(r)) \cdot \big[u(w_1(r), q_1) + v(w_1(r), r)\big] - c\big(e_1(w_1(r))\big) - \underline{u}$ for all $r \in [0,1]$, with $e_1(w) = e_1^*(w)$. In other words, membership prices for $\mathscr{F}_1$ equal the surplus of each type. If I involves social exclusion, then for $\mathscr{F}_1$, this is IC with prices $p_1(r)$, using the argument from $|I| = 1$. Let $e_2(w) = e_2^*(w)$. By Assumption 3, $e_2^*(\underline{w}_2) \cdot \big[u(\underline{w}_2, q_2) + v(\underline{w}_2, 0)\big] - c(e_2^*(\underline{w}_2)) \geq \underline{u}$. We can thus find $p_2(0) > 0$ such that $U(\underline{w}_2, 2, \mathscr{F}_2) = \underline{u} = U(\underline{w}_2, 1, \mathscr{F}_1)$. As $u(w, q_2) + v(w, 0)$ is strictly increasing in w (Assumption 1), $U(w, 2, \mathscr{F}_2) \leq \underline{u}$ for all $w < \underline{w}_2$. IC holds for all $w \leq \underline{w}_2$. By joining $\mathscr{F}_1$, any type $w > \underline{w}_2$ obtains:

$$\begin{aligned}
&e_1(w) \cdot \big[u(w, q_1) + v(w, 1)\big] - c(e_1(w)) - p_1(1)\\
=&e_1(w) \cdot \big[u(w, q_1) + v(w, 1)\big] - c(e_1(w)) - \Big(e_1(\underline{w}_2) \cdot \big[u(\underline{w}_2, q_1) + v(\underline{w}_2, 1)\big] - c(e_1(\underline{w}_2)) - \underline{u}\Big).
\end{aligned}$$

IC thus further requires:

$$p_2(r_2(w)) \leq e_1(w) \cdot \big[u(w, q_1) + v(w, 1)\big] - c(e_1(w)) - \Big(e_1(\underline{w}_2) \cdot \big[u(\underline{w}_2, q_1) + v(\underline{w}_2, 1)\big] - c(e_1(\underline{w}_2)) - \underline{u}\Big).$$

We can then define an incentive compatible $p_2(r)$ as follows:

$$\begin{aligned}
p_2(r_2(w)) = p_2(0) + \int_{\underline{w}_2}^{w} \frac{\partial}{\partial x}\Big[&e_2(x) \cdot \big[u(x, q_2) + v(x, r_2(x))\big] - c(e_2(x))\\
&-\Big(e_1(x) \cdot \big[u(x, q_1) + v(x, 1)\big] - c(e_1(x))\Big)\Big]\, dx,
\end{aligned}$$

where $p_2(0) = e_2(\underline{w}_2)\cdot\left[u(\underline{w}_2,q_2)+v(\underline{w}_2,0)\right]-c(e_2(\underline{w}_2))-\underline{u}$. Substituting in for $p_2(0)$ and using the definition of $p_1(1)$, the revenue generated by group $\mathscr{F}_2$ equals:

$$\begin{aligned}\Big(F(\overline{w})-F(\underline{w}_2)\Big)\cdot\Big(p_1(1)+\int_{\underline{w}_2}^{\overline{w}_2} & e_2(w)\cdot\left[u(w,q_2)+v(w,r_2(w))\right]-c(e_2(w))\\ & -\Big(e_1(w)\cdot\left[u(w,q_1)+v(w,1)\right]-c(e_1(w))\Big)\,dF_2(w)\Big).\end{aligned} \tag{12}$$

As $p_1(1) = e_1(\underline{w}_2)\cdot\left[u(\underline{w}_2,q_1)+v(\underline{w}_2,1)\right]-c(e_1(\underline{w}_2))-\underline{u}$, and as $v(w,r)$ (and hence also $e_2(w)$) is increasing in r, C1 ensures (12) is weakly greater than 0. Since revenue from $\mathscr{F}_1$ is also positive, overall revenue is positive. $(I,\mathbf{e},\mathbf{p}^r)$ is an equilibrium group provision. □

Example 3.2 (Section 5.1) demonstrates that this does not extend to a larger number of groups, unless stronger assumptions are made. C1* provides a (much) stronger stronger version of C1 that allows the provision of a partition for any U (Proposition 9). This stronger condition is always satisfied if status concern is equal across types, but limits the set of partitions that can be achieved in equilibrium with strict complementarity. It might not be satisfied by either the (unconstrained) welfare or profit maximising partition.

Condition C1*. *An interval partition I satisfies C1* if there exist $e_k(w) = e_k^*(w)$, for all $k \in A\backslash\{\emptyset\}$ and $w \in [\underline{w}_1,\overline{w}]$, and corresponding $\{\mathscr{F}_k\}$, such that for any two $\mathscr{F}_l,\mathscr{F}_h \in \{\mathscr{F}_k\}$ with $\underline{w}_l \le \overline{w}_h$, and all $w \in [\underline{w},\overline{w}]$:*

$$\frac{\partial}{\partial w}\Big[\overline{e}_h(w)\big[\underline{e}_h(w)\cdot\left[u(w,q_h)+v(w,0)\right]-c(\underline{e}_h(w))-\Big(\overline{e}_l(w)\cdot\left[u(w,q_l)+v(w,1)\right]-c(\overline{e}_l(w))\Big)\Big]\ge 0,$$

where $\overline{e}_l(w)$ is the optimal engagement of type w, for q_l and $r=1$, and $\underline{e}_h(w)$ for q_h and $r=0$.

If C1* holds, then the complementarity in w and q outweighs any (possible) negative effects from the interaction between w and r. It extends Assumption 2 to cases where a higher quality group delivers lower utility for some members. Lemma 6 shows that C1* is indeed a strengthening of C1. C1* ensures transfers needed for IC decline with type and can thus be achieved in a budget-balanced way (Proposition 9).

Lemma 6. *If an interval partition I satisfies C1*, then it satisfies C1.*

Proof. Suppose $q_h > q_l$. It follows from C1* that

$$\int_{\underline{w}_h}^{\overline{w}_h}\int_{\underline{w}_h}^{w}\frac{\partial}{\partial x}\Big(\underline{e}_h(x)\cdot\left[u(x,q_h)+v(x,0)\right]-c(\underline{e}_h(x))-\hat{U}(x,l,\mathscr{F}_l)\Big)\,dx\,dF_h(w)\ge 0.$$

This can be rewritten as:

$$\int \underline{e}_h(x)\cdot\left[u(x,q_h)+v(x,0)\right]-c(\underline{e}_h(x))-\hat{U}(w,l,\mathscr{F}_l)\,dF_h(w)\ge \hat{U}(\underline{w}_h,h,\mathscr{F}_h)-\hat{U}(\underline{w}_h,l,\mathscr{F}_l),$$

and hence by Assumption 3:

$$\int \underline{e}_h(x)\cdot\left[u(x,q_h)+v(x,0)\right]-c(\underline{e}_h(x))-\hat{U}(w,l,\mathscr{F}_l)\,dF_h(w)\ge \underline{u}-\hat{U}(\overline{w}_l,l,\mathscr{F}_l).$$

As $v(w,r)$ is strictly increasing in r (Assumption 1), we can conclude that:

$$\int e_h(w)\cdot\left[u(w,q_h)+v(w,r_h(w))\right]-c(e_h(w))-\hat{U}(w,l,\mathscr{F}_l)\Big]\,dF_h(w)$$
$$\geq \underline{u}-\hat{U}(\overline{w}_l,l,\mathscr{F}_l),$$

which corresponds to C1. □

Proposition 9. *For any interval partition I of some $[w_1,\overline{w}]\subseteq[\underline{w},\overline{w}]$, if I satisfies C1*, then there exist $\mathbf{p}^r$ and $\mathbf{e}$, such that $(I,\mathbf{e},\mathbf{p}^r)$ is an equilibrium group provision.*

Proof. By Assumption 1, C1* can only hold if there exists $e_k^*(w)$, for all $k\in A$, such that $\underline{w}_h\geq\overline{w}_l\implies q_h>q_l$. WLOG, assume that social groups are ordered such that $q_1<q_2<...<q_n$. It follows from Lemma 6 and Proposition 8 that the statement is true for $|I|\leq 2$. Suppose $|I|>2$. Let $p_1(r)$ and $p_2(r)$ be defined as in the proof of Proposition 8. Define the price $p_k(r)$ of group $\mathscr{F}_k$, with $k>2$, as follows:

$$\begin{aligned}p_k(r_k(w))=&e_k(\underline{w}_k)\cdot\left[u(\underline{w}_k,q_k)+v(\underline{w}_k,0)\right]-c(e_k(\underline{w}_k)-\underline{u}\\&-\int_{\underline{w}_2}^{\underline{w}_k}\frac{\partial}{\partial w}\Big(e_{g(w)-1}(w)\cdot\left[u(w,q_{g(w)-1})+v(w,1)\right]-c(e_{g(w)-1}(w))\Big)dw\\&+\int_{\underline{w}_k}^{w}\frac{\partial}{\partial w}\Big(\hat{U}(w,k,\mathscr{F}_k)-\hat{U}(w,k-1,\mathscr{F}_{k-1})\Big)dw,\end{aligned}$$

where $w\in[\underline{w}_k,\overline{w}_k]$, and $e_k(w)=e_k^*(w)$. It is easily verified that for all $\mathscr{F}_k$ with $k>1$, $\hat{U}(\underline{w}_k,k,\mathscr{F}_k)-p_k(0)=\hat{U}(\underline{w}_k,k-1,\mathscr{F}_{k-1})-p_{k-1}(1)$, meaning $\mathbf{p}^r$ ensures indifference at the cut-off. It remains to be shown that IC holds for all other types, and revenues are non-negative.
First, it is shown that all types weakly prefer their group to all lower quality groups ('downward IC'). Observe that the utility a type w, with $g(w)=k$, obtains from joining some $l\leq k$, equals:

$$\begin{aligned}&e_l(w)\cdot\left[u(w,q_l)+v(w,1)\right]-c(e_l(w))-p_l(1)\\=&\int_{\underline{w}_2}^{w}\frac{\partial}{\partial w}\Big(e_{\gamma(w)}(w)\cdot\left[u(w,q_{\gamma(w)})+v(w,1)\right]-c(e_{\gamma(w)}(w))\Big)dw,\end{aligned}$$

where $\gamma(w)=\max\{g(w)-1,l\}$. As type and quality are complements, this is maximised for $l=k$. Downward IC is satisfied. Next, it is shown that all types weakly prefer their group over all higher quality groups ('upward IC'). Consider a type w, with $g(w)=k$, joining a group $h>k$. They obtain utility $\hat{U}(w,h,\mathscr{F}_h)-p_h(0)$, while type $\underline{w}_h$ joining the same group obtains $\hat{U}(\underline{w}_h,h,\mathscr{F}_h)-p_h(0)$. The difference in utility from membership in $\mathscr{F}_h$ between both types thus equals:

$$\int_{w}^{\underline{w}_h}\frac{\partial}{\partial x}\Big(e_h(x)\cdot\left[u(x,q_h)+v(x,0)\right]-c(e_h(x))\Big)dx. \tag{13}$$

By definition of $\mathbf{p}^r$, the net utility difference between both types, when following g, equals:

$$\int_{w}^{\underline{w}_h}\frac{\partial}{\partial w}\Big(e_{g(x)-1}(x)\cdot\left[u(x,q_{g(x)-1})+v(x,1)\right]-c(e_{g(x)-1}(x))\Big)dx. \tag{14}$$

By C1*, (13) is weakly greater than (14). This implies that if any upward deviation is beneficial and hence $\hat{U}(w,h,\mathscr{F}_h) - p_h(0) > \hat{U}(w,k,\mathscr{F}_k) - p_k(r_k(w))$, then $\hat{U}(\underline{w}_h, h, \mathscr{F}_h) - p_h(0) > \hat{U}(\underline{w}_h, h-1, \mathscr{F}_{h-1}) - p_{h-1}(1)$. But this contradicts indifference at the cut-off, which is ensured by construction of $\mathbf{p}^r$. Upward IC is also satisfied.

Finally, to show that the sum of membership payments is non-negative, note that $p_1(1) > p_1(0) \geq 0$. Furthermore,

$$\begin{aligned} p_k(r) - p_1(1) = &\int_{\underline{w}_2}^{\underline{w}_k} \frac{\partial}{\partial w}\Big(\underline{e}_{g(w)}(w)\cdot\big[u(w,q_{g(w)} + v(w,0)\big] - c(\underline{e}_{g(w)}(w)) - \hat{U}(w,g(w)-1,\mathscr{F}_{g(w)-1})\Big)dw \\ &+ \int_{\underline{w}_k}^{r_k^{-1}(r)} \frac{\partial}{\partial w}\Big(\hat{U}(w,k,\mathscr{F}_k) - \hat{U}(w,k-1,\mathscr{F}_{k-1})\Big)dw, \end{aligned} \tag{15}$$

where again $\underline{e}_{g(w)}(w)$ is the optimal engagement for a type w given $q_{g(w)}$ and $r = 0$. By C1*:

$$\frac{\partial}{\partial w}\Big(\underline{e}_{g(w)}(w)\cdot\big[u(w,q_{g(w)} + v(w,0)\big] - c(\underline{e}_{g(w)}(w)) - \hat{U}(w,g(w)-1,\mathscr{F}_{g(w)-1})\Big) \geq 0,$$

which implies both terms of (15) are non-negative. Prices, and thus revenue, are non-negative.

□

A.5 Proofs (Section 5)

Proof of Proposition 6. The first part relating to $\mathbf{p}^w$ follows from Lemma 5. To prove the second part, let $\mathbf{e}$ be such that $e_k(w) = e_k^*(w)$, for all $w \in [\underline{w}_1, \overline{w}]$ and $k \in A \setminus \{\emptyset\}$. Existence of such equilibrium e_k^* and hence q_k follows from Lemma 4. The remainder of the proof focuses on constructing prices $\mathbf{p}^r$ that ensure IC and non-negative revenue given I. For every group $k \in A \setminus \{\emptyset\}$, let prices be as follows:

$$p_k(r) = \begin{cases} \hat{U}(w_1(r),1,\mathscr{F}_1) - \underline{u} & \text{if } k = 1 \\ p_{k-1}(1) + \hat{U}(\underline{w}_k, k, \mathscr{F}_k) - \hat{U}(\underline{w}_k, k-1, \mathscr{F}_k - 1) & \\ + \int_{\underline{w}_k}^{w_k(r)} \frac{\partial}{\partial w}\big(\hat{U}(w,k,\mathscr{F}_k) - \hat{U}(w,k-1,\mathscr{F}_{k-1})\big)dw & \text{if } k > 1, \end{cases}$$

where $w_k(r) \equiv r_k^{-1}(r)$, i.e., the type w such that $r_k(w) = r$, given I. This leaves members of group $\mathscr{F}_1$ with $\underline{u}$ and all others with their benefit from a possible downward deviation, thus satisfying downward IC. Upward IC is ensured by complementarity in w and q of $u(w,q)$, as well as $v(r)$ being independent of type. IC thus holds by construction.

Non-negative revenue is shown by induction. Revenue from $\mathscr{F}_1$ is trivially positive since $\hat{U}(w_1(r),1,\mathscr{F}_1) - \underline{u} \geq 0$, for all r. Since $\hat{U}(\underline{w}_2, 2, \mathscr{F}_2) \geq \underline{u}$ and $p_1(1) = \hat{U}(\underline{w}_2, 1, \mathscr{F}_1)$, it follows that $p_2(0) \geq 0$. Recall that $\hat{U}(w,k,\mathscr{F}_k) = e_k^*(w)\cdot\big[u(w,q_k) + v(r_k(w))\big] - c(e_k^*(w))$. Since $q_k \geq q_{k-1}$ (monotonic quality), and hence $u(w,q_k) - u(w,q_{k-1}) > u(w',q_k) - u(w',q_{k-1})$ for all $w > w'$, it follows from $v(r)$ being equal for all types and the proof of Lemma 3 that:

$$\frac{\partial}{\partial w}\Big(\underline{e}_k(w)\cdot\big[u(w,q_k)+v(0)\big]-c(\underline{e}_k(w))-\big(e^*_{k-1}(w)\cdot\big[u(w,q_{k-1})+v(1)\big]-c(e^*_{k-1}(w))\big)\Big)\geq 0,$$

where $\underline{e}_k(w)$ is the optimal engagement for a type w given q_k and (hypothetical) rank 0. This implies $\int_0^r \frac{\partial}{\partial w}\big(\hat{U}(w_k(r),k,\mathscr{F}_k)-\hat{U}(w_{k-1}(r),k-1,\mathscr{F}_{k-1})\big)dr\geq 0$ for all $r>0$, i.e., prices must be increasing in r. Accordingly, $p_2(r)\geq 0$, for all $r\in[0,1]$.

For the induction step, suppose $p_k(r)\geq 0$ for all $r\in[0,1]$ and $k\in\{1,...,m-1\}$. Since $q_m\geq q_{m-1}\geq ...\geq q_1$ and status concern is equal across types,

$$\hat{U}(\underline{w}_m,m,\mathscr{F}_m)-\hat{U}(\underline{w}_m,m-1,\mathscr{F}_{m-1})\geq\hat{U}(\underline{w}_{m-1},m-1,\mathscr{F}_{m-1})-\hat{U}(\underline{w}_{m-1},m-2,\mathscr{F}_{m-2}).$$

Note that $p_{m-1}(0)=p_{m-2}(1)+\hat{U}(\underline{w}_{m-1},m-1,\mathscr{F}_{m-1})-\hat{U}(\underline{w}_{m-1},m-2,\mathscr{F}_{m-2})$. By the induction hypothesis $p_{m-1}(1)\geq p_{m-1}(0)\geq 0$. It follows that $p_m(0)\geq 0$ and hence $p_m(r)\geq 0$ for all $r\in[0,1]$. Revenue from group $\mathscr{F}_m$ is non-negative. The result follows. □

Proof of Proposition 7. Suppose there are n groups and assume wlog that they are ordered such that $\overline{w}_1<\overline{w}_2<...<\overline{w}_n$. For each $i\in\{1,...,n\}$, let $e_i(w)=\min\{e_i^*(w),\overline{e}_i\}$, where as before $e_i^*(w)$ solves the agent's engagement optimisation problem for group $\mathscr{F}_i$.

The proof is by induction. For each $i\in\{1,...,n\}$, define the following function that describes the utility from group membership in $\mathscr{F}_i$ (given $\overline{\boldsymbol{e}}$, and hence q_i) net of prices for some e: $\hat{U}_i(w,e)\equiv e\big[u(w,q_i)+v(w,r_i(w))\big]-c(e)$. Consider group $\mathscr{F}_n$ and the adjacent $\mathscr{F}_{n-1}$. Upward IC requires that for all $w\in\mathscr{F}_{n-1}$, the following holds:

$$p_n-p_{n-1}\;\geq\;\hat{U}_n\big(w,e_n(w)\big)-\hat{U}_{n-1}\big(w,e_{n-1}(w)\big). \tag{16}$$

Suppose in group $\mathscr{F}_n$, the engagement limit is $\overline{e}_n>0$ (which might not be binding). Let $\hat{e}_n=\min\big\{e_n^*(\underline{w}_{n-1})\ ,\ \overline{e}_n\big\}$. Note that if $\hat{e}_n$ was the engagement limit for $\mathscr{F}_n$, it would be binding for all types $w\geq\underline{w}_{n-1}$. As $u(w,q)+v(w,0)$ is strictly increasing in w (Assumption 1), we can find $\overline{e}_{n-1}\leq\min\{\hat{e}_n,e_{n-1}(\underline{w}_{n-1})\}$ (i.e., a limit that is binding for all group members and, by monotonic quality, $q_{n-1}\leq q_n$) such that for almost all $w\in[\underline{w}_{n-1},\overline{w}_{n-1}]$ the following two hold:

$$\hat{U}_{n-1}\big(\underline{w}_{n-1},\overline{e}_{n-1}\big)<\hat{U}_n\big(\underline{w}_{n-1},\hat{e}_n\big), \tag{17}$$

$$\frac{\partial}{\partial w}\hat{U}_{n-1}(w,\overline{e}_{n-1})<\frac{\partial}{\partial w}\hat{U}_n(w,\hat{e}_n). \tag{18}$$

To see that such an $\overline{e}_{n-1}$ exists, note that $\lim_{e\to 0}\hat{U}_{n-1}(w,e)=\lim_{e\to 0}\frac{\partial}{\partial w}\hat{U}_{n-1}(w,e)=0$ for all $w\in[\underline{w},\overline{w}]$ and any $q_{n-1}\geq\underline{q}$, while both $\hat{U}_n\big(\underline{w}_{n-1},\hat{e}_n\big)$ and $\frac{\partial}{\partial w}\hat{U}_n\big(w,\hat{e}_n\big)$ are strictly positive. It follows that for all $w\in[\underline{w}_{n-1},\overline{w}_{n-1})$,

$$\hat{U}_n\big(\overline{w}_{n-1},\hat{e}_n\big)-\hat{U}_n\big(w,\hat{e}_n\big)>\hat{U}_{n-1}\big(\overline{w}_{n-1},\overline{e}_{n-1}\big)-\hat{U}_{n-1}\big(w,\overline{e}_{n-1}\big),$$

and hence

$$\hat{U}_n\big(\overline{w}_{n-1}, e_n(\overline{w}_{n-1})\big) - \hat{U}_n\big(w, e_n(w)\big) > \hat{U}_{n-1}\big(\overline{w}_{n-1}, \overline{e}_{n-1}\big) - \hat{U}_{n-1}\big(w, \overline{e}_{n-1}\big),$$

noting that $\hat{U}_n(\overline{w}_{n-1}, e_n(w)) - \hat{U}_n(w, e_n(w)) \geq \hat{U}_n(\overline{w}_{n-1}, \hat{e}_n) - \hat{U}_n(w, \hat{e}_n)$, for all $w \leq \overline{w}_{n-1}$, and $\hat{U}_n(w', e_n(w')) \geq \hat{U}_n(w', e_n(w))$, for all $w' > w$. Let $p_n = p_{n-1} + \hat{U}_n(\overline{w}_{n-1}, e_n(\overline{w}_{n-1})) - \hat{U}_{n-1}(\overline{w}_{n-1}, \overline{e}_{n-1})$, which is necessary for IC (Lemma 2). By construction of $\overline{e}_{n-1}$ and p_n, $p_n - p_{n-1} \geq \hat{U}_n\big(w, e_n(w)\big) - \hat{U}_{n-1}\big(w, \overline{e}_{n-1}\big)$, for all w in $\mathscr{F}_{n-1}$, meaning (16) holds and thus upward IC is satsfied. Furthermore, as $p_n - p_{n-1} > 0$, downward IC for all w in $\mathscr{F}_n$ follows from Assumption 2.

For the induction step, consider a group $\mathscr{F}_{i-1}$ and suppose IC is satisfied for all groups $j \geq i$. We can repeat the previous argument with the modification that (18) is replaced by:

$$\frac{\partial}{\partial w}\hat{U}_{i-1}(w, \overline{e}_{i-1}) < \frac{\partial}{\partial w}\hat{U}_i(w, \hat{e}_i), \quad \forall w \in [\underline{w}_{i-1}, \overline{w}_{i-1}).$$

It follows from the previous argument that this implies IC between $\mathscr{F}_{i-1}$ and $\mathscr{F}_i$, and, in fact, downward IC for all $j \geq i$ (noting that IC is satisfied between all groups $j \geq i$.

As $r_j(w) = 0$ and $q_j \geq q_i$, for all $w \in [\underline{w}_{i-1}, \overline{w}_{i-1}]$ and $j \geq i$, it further follows that:

$$\frac{\partial}{\partial w}\hat{U}_{i-1}(w, \overline{e}_{i-1}) < \min\Big\{\frac{\partial}{\partial w}\hat{U}_j(w, \hat{e}_j)\Big\}_{j=i}^{n}, \quad \forall w \in [\underline{w}_{i-1}, \overline{w}_{i-1}).$$

Accordingly, upward IC is satisfied for all groups $j \geq i$. The result follows. □

Proof of Corollary 7.1. Suppose $(I, \overline{\boldsymbol{e}}, \mathbf{p})$ is an equilibrium provision that involves social exclusion. Let $I' = \{[\underline{w}, \underline{w}_1]\} \cup I$, and let $\mathscr{F}_0$ denote the added social group for some $e_0(w)$. It follows from the proof of Proposition 7 that since $(I, \overline{\boldsymbol{e}}, \mathbf{p})$ is an equilibrium, there exist $\mathbf{p}'$ and $\overline{\boldsymbol{e}}'$, with $e_0(w) > 0$ and $e'_k(w) = e_k(w)$, for all $k \neq 0$, such that $(I', \overline{\boldsymbol{e}}', \mathbf{p}')$ is an equilibrium provision. Clearly, any such $(I', \overline{\boldsymbol{e}}', \mathbf{p}')$ achieves strictly higher welfare. □

A.6 Additional Result (Section 6)

Proposition 10. *Suppose quality is equal to the mean type, i.e., $q_k = \int wF_k(w)$. Then for any distribution of types F, there exists a mean-preserving contraction F' such that any equilibrium group provision under F' consist of a single group, meaning $|I| = 1$.*

Proof. Let w^* be the mean type of F and let F' be a uniform distribution over $[w^* - \frac{\epsilon}{2}, w^* + \frac{\epsilon}{2}]$, with $\epsilon > 0$. For ϵ small enough, F' is a mean-preserving contraction of F. The quality difference between any two groups is bounded by ϵ. Let $\underline{q} = w^* - \frac{\epsilon}{2}$ and $\overline{q} = w^* + \frac{\epsilon}{2}$, i.e., the strict lower- and upper-bound on quality given F'. Let $\Delta V(\epsilon) = v(w^* - \frac{\epsilon}{2}, 1) - v(w^* - \frac{\epsilon}{2}, 0)$, i.e., the lowest utility difference between rank 1 and 0 given F'. Furthermore, let $\Delta U(\epsilon) = u(w^* + \frac{\epsilon}{2}, \overline{q}) - u(w^* + \frac{\epsilon}{2}, \underline{q})$, i.e., the maximum difference in utility derived from the quality of two different groups. Note that $\lim_{\epsilon \to 0} \Delta U(\epsilon) = 0$, while $\lim_{\epsilon \to 0} \Delta U(\epsilon) = v(w^*, 1) - v(w^*, 0) > 0$. It follows that there exists F' such that $u(w,\overline{q}) + v(w,0) < u(w,\underline{q}) + v(w,1)$, for all w in the support of F'. For such an F', IC is violated for any group structure with more than one group. □

A.7 Positive Welfare Effects of Status Concern

This section formalises the observation in Example 1.4 that welfare can be higher if individuals exhibit stronger status concern. It presents sufficient conditions for welfare to be higher when individuals attach greater weight to status than the planner ($\lambda < \alpha$). In other words, despite welfare being measured according to U_λ, choices based on U_α can lead to a superior outcome. It follows from Proposition 3 that such a positive welfare effect can only arise if engagement affects quality. For simplicity, it is assumed here that quality is entirely determined by the average engagement in a group (subject to an upper and lower bound). More specifically,

$$q_k = \max\left\{\min\left\{\overline{q}, \phi\left(\int e_k(w)\mathrm{d}F_k(w)\right)\right\}, \underline{q}\right\}, \tag{19}$$

where $\underline{q} < \overline{q}$, and ϕ some suitable, strictly increasing and differentiable function. For preferences U_α, higher α (weakly) limits the ability of a planner to sort individuals and affects their engagement choice. Due the positive externality created by engagement, a positive welfare effect obtains for some $\alpha > \lambda$ if the constraint on sorting is not binding and status concern increases engagement for a sufficiently large fraction of the population. Condition C2 guarantees that latter is the case, i.e., higher α increases engagement on average. Furthermore, by requiring no two groups to have the same quality, it also generically ensures that the constraint on sorting is not binding. This is, for instance, always the case for $\lambda = 0$, since all groups having distinct quality implies that no set (of positive measure) of agents can be exactly indifferent between two groups.

Condition C2. *A group structure $(I, \mathbf{e}, \mathbf{p})$ satisfies C2, if for any $k \in A \setminus \{\emptyset\}$ and $\tilde{r} \in [0, \frac{1}{2})$:*

$$\max\{0, -v(w_k(\tilde{r}), \tilde{r}) + u(w_k(\tilde{r}, q_k)\} \le v(w_k(1-\tilde{r}), 1-\tilde{r}) - u(w_k(1-\tilde{r}), q_k),$$

where $w_k(\tilde{r}) \equiv r_k^{-1}(\tilde{r})$, and $q_k \neq q_l$, for all $k, l \in A$, with at least one $\mathscr{F}_k$ such that $q_k \in (\underline{q}, \overline{q})$.

Loosely speaking, C2 holds if the incentive status concern provides to raise engagement for higher ranks (above median types) outweighs any potential negative effects on lower types. Furthermore, such a change in engagement must affect quality, i.e., there is at least one group with quality strictly inside the lower and upper bound. Proposition 11 and Corollary 11.1 then show that, given (19) and quadratic engagement costs, C2 is (generically) sufficient for welfare to be higher for some $\alpha' > \lambda$, than for $\alpha = \lambda$, and specifically for when the planner attaches no weight to status, i.e., $\alpha' > \lambda = 0$.

Proposition 11. *Suppose an equilibrium group provision $(I, \mathbf{e}, \mathbf{p})$, for $\alpha = \lambda$, satisfies C2 and quality is as in* (19)*. Then there generically exists an equilibrium group provision $(I, \mathbf{e}', \mathbf{p}')$, for $\alpha' > \lambda$, that achieves higher welfare (as evaluated by U_λ).*

Proof. **Step 1:** It is first shown that as α increases, average engagement, and hence group quality, increase if $C2$ holds. Let $\mathscr{U}_w(\alpha) \equiv (1-\alpha)\cdot u(w,q_k) + \alpha \cdot v(w, r_k(w))$, where $k = g(w)$, and g the assignment function corresponding to I. A marginal change in α affects $\mathscr{U}_w$ as follows: $\frac{\partial}{\partial \alpha}\mathscr{U}_w(\alpha) = -u(w,q_k) + v(w, r_k(w))$. Let $\mathscr{U}_w(\alpha'|\alpha) = \left[1+\alpha' \cdot \frac{-u(w,q_k)+v(w,r_k(w))}{\mathscr{U}_w(\alpha)}\right] \cdot \mathscr{U}_w(\alpha)$, which can be obtained from a Taylor expansion at α, keeping q_k constant. Take any $\tilde{w}$, with $\tilde{r} = r_k(\tilde{w}) \in [0, \frac{1}{2})$, and $\hat{w}$, with $r_k(\hat{w}) = 1-\tilde{r}$. It follows from C2 that :

$$
\begin{aligned}
\mathscr{U}_{\tilde{w}}(\alpha'|\alpha) + \mathscr{U}_{\hat{w}}(\alpha'|\alpha) \quad &= \quad \left[1+\alpha' \cdot \frac{-u(\tilde{w},q_k)+v(\tilde{w},\tilde{r})}{\mathscr{U}_{\tilde{w}}(\alpha)}\right]\cdot \mathscr{U}_{\tilde{w}}(\alpha) \\
&\quad + \left[1+\alpha' \cdot \frac{-u(\hat{w},q_k)+v(\hat{w},1-\tilde{r})}{\mathscr{U}_{\hat{w}}(\alpha)}\right]\cdot \mathscr{U}_{\hat{w}}(\alpha) \qquad (20) \\
&\geq \quad \mathscr{U}_{\tilde{w}}(\alpha) + \mathscr{U}_{\hat{w}}(\alpha).
\end{aligned}
$$

Let $e^*_w(\alpha)$ be the optimal engagement for type w given $\mathscr{U}_w(\alpha)$ and cost function $c(e)$, and accordingly for $\mathscr{U}_w(\alpha'|\alpha)$. As $c(e)$ is quadratic (Assumption 5), $e^*_w(\alpha) \propto \mathscr{U}_w(\alpha)$. It then follows from (20) that $e^*_{\tilde{w}}(\alpha'|\alpha) + e_{\hat{w}}(\alpha'|\alpha)^* \geq e^*_{\tilde{w}}(\alpha) + e^*_{\hat{w}}(\alpha)$. Accordingly, $\int_0^1 e^*_{w_k(\tilde{r})}(\alpha'|\alpha)d\tilde{r} = \int_0^{\frac{1}{2}} e^*_{w_k(\tilde{r})}(\alpha'|\alpha) + e^*_{w_k(1-\tilde{r})}(\alpha'|\alpha)d\tilde{r} \geq \int_0^1 e^*_{w_k(\tilde{r})}(\alpha)d\tilde{r}$, with $w_k(\tilde{r}) \equiv r_k^{-1}(\tilde{r})$. It then follows from (19) that $q'_k \geq q_k$ with the inequality strict for at least one group. Denote the corresponding provision by $(I, \mathbf{e}', \mathbf{p}')$, where $\mathbf{p}'$ are set according Lemma 2.

If for $(I, \mathbf{e}, \mathbf{p})$ IC holds strictly for all $w \in (\underline{w}_k, \overline{w}_k)$, and $k \in A \setminus \{\emptyset\}$, then it follows from continuity in α, e, and q that this also applies to α' sufficiently close to α. It follows that the corresponding $(I, \mathbf{e}', \mathbf{p}')$ is an equilibrium provision. If, in contrast, there exists a subset $\mathscr{W}_{k,l} \subseteq (\underline{w}_k, \overline{w}_k)$ that is indifferent between k and l, then - given that $q_k \neq q_l$, for all distinct $k, l \in A$ - there exists an ϵ-perturbation in α and/or F such that IC holds strictly for all $w \in (\underline{w}_k, \overline{w}_k)$ and a corresponding equilibrium provision $(I, \mathbf{e}, \mathbf{p})$. Generically, there exist $\alpha' > \alpha$ such that $(I, \mathbf{e}', \mathbf{p}')$ is an equilibrium provision.

Step 2: To complete the argument, it is shown that for α' sufficiently close to α, an increase in group quality through higher engagement increases welfare. Individual equilibrium engagement choices satisfy $u(w,q) + v(w,r) = \frac{\partial}{\partial e}c(e)$, which implies engagement is inefficiently low as it ignores the effect on quality. From the perspective of the planner with $\lambda = \alpha$, a change in α affects the utility of all individuals with a particular type w (through the change in optimal engagement) as follows:

$$
\begin{aligned}
\frac{\partial}{\partial \alpha} U_\alpha(w,k,\mathscr{F}_k) &= \left[u(w,q) + v(w,k) + \frac{\partial}{\partial q}u(w,q_k)\frac{\partial}{\partial \tilde{e}}\phi(\tilde{e}_k(\alpha)) - \frac{\partial}{\partial e^*}c(e^*_w(\alpha))\right]\frac{\partial}{\partial \alpha}\tilde{e}_k(\alpha) \\
&= \frac{\partial}{\partial q}u(w,q_k)\frac{\partial}{\partial \tilde{e}}\phi(\tilde{e}_k(\alpha)) f_k(w) \frac{\partial}{\partial \alpha} e^*_w(\alpha),
\end{aligned}
$$

where the second equality follows from the Envelope Theorem, and $\tilde{e}_k(\alpha) \equiv \int e^*_w(\alpha) \mathrm{d}F_k(w)$.

Integrating over all individuals in a group $\mathscr{F}_k$ yields the marginal change in welfare (at α):

$$\begin{aligned}\int \frac{\partial}{\partial\alpha} U_\alpha(w,k,\mathscr{F}_k)\mathrm{d}F_k(w) >& \frac{\partial}{\partial q} u(\underline{w}_k, q_k)\frac{\partial}{\partial\tilde{e}}\phi(\tilde{e}_k(\alpha))\int \frac{\partial}{\partial\alpha} e^*_w(\alpha)\mathrm{d}F_k(w)\\ =& \frac{\partial}{\partial q} u(\underline{w}_k, q_k)\frac{\partial}{\partial\tilde{e}}\phi(\tilde{e}_k(\alpha))\int_0^1 \frac{\partial}{\partial\alpha} e^*_{w_k(r)}(\alpha)\mathrm{d}r \geq 0,\end{aligned}$$

where the first inequality follows from Step 1 and $\frac{\partial}{\partial\tilde{e}}\phi(\tilde{e}) > 0$, the equality follows from the probability integral transform, and the final inequality follows from the previous argument. We can conclude that there exists an interval $(\alpha, \overline{\alpha})$, such that for all $\alpha' \in (\alpha, \overline{\alpha})$, the corresponding $(I, \mathbf{e}', \mathbf{p}')$ achieves higher welfare than $(I, \mathbf{e}, \mathbf{p})$. □

Corollary 11.1. *Suppose an equilibrium group provision* $(I, \mathbf{e}, \mathbf{p})$*, for* $\alpha = \lambda = 0$*, satisfies C2 and quality is as in* (19)*. Then there exists an equilibrium group provision* $(I, \mathbf{e}', \mathbf{p}')$*, for* $\alpha' > 0$*, that achieves higher welfare.*

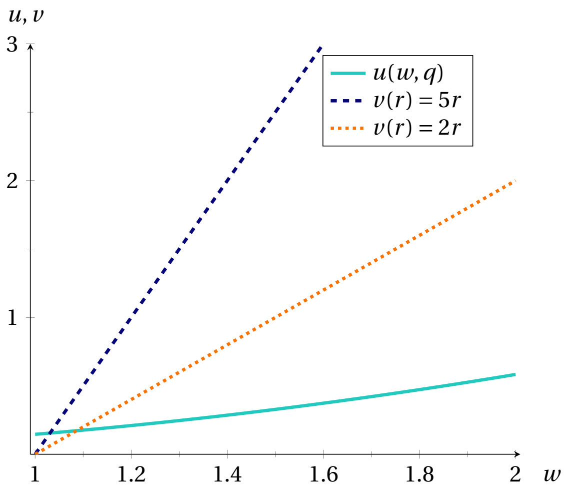
Proof. As C2 is satisfied, no two groups $k, l \in A\setminus\{\emptyset\}$ have identical quality. It follows from strict complementarity in type and quality, and equilibrium prices (Lemma 2) that in an equilibrium $(I, \mathbf{e}, \mathbf{p})$, for all $k \in A\setminus\{\emptyset\}$, any type $w \in (\underline{w}_k, \overline{w}_k)$ strictly prefers k to any other $l \in A$. Continuity in α, e, and q implies that for any $\alpha' \in [0, \overline{\alpha})$ and $\overline{\alpha}$ sufficiently close to 0, this also holds for $U_{\alpha'}$ with corresponding provision $(I, \mathbf{e}', \mathbf{p}')$. The result then follows from Proposition 11. □

Example 4.1 demonstrates the previous results in a setting where status concern is constant across types and parametrised by a scaling factor κ. A higher κ signifies a higher 'return' from status for (almost) all ranks, while - as before - a higher α represents 'stronger' status concern. For low κ, status provides an insufficient incentive for (most) individuals to engage more with the group. This is particularly the case for larger $\overline{w}$ since higher types lead to higher group quality at any $\alpha < 1$. Condition C2 is not satisfied and welfare (evaluated at $\lambda = 0$) is, in fact, decreasing in α. For larger κ, C2 holds and, in line with the results, engagement, quality and welfare are increasing in α for at least some α and $\lambda = 0$. Welfare is higher with status concern, even if the welfare criterion attaches no weight to v. For even larger κ, however, the range of α for which there is a positive effect is decreasing. The incentive status provides is so strong that individuals over-invest in engagement; the cost of engagement negates the positive effects from social spillovers.

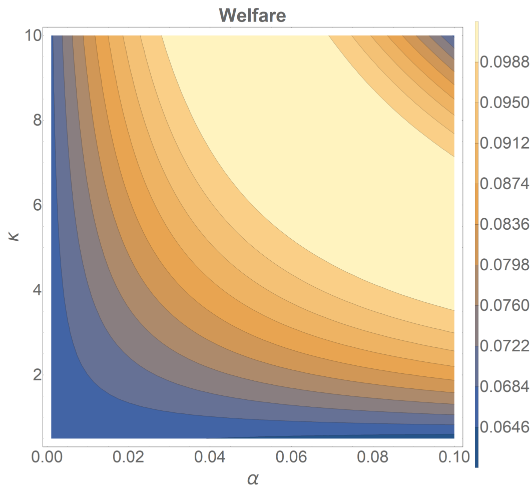
Example 4.1. Types are distributed uniformly over $[1, \overline{w}]$, utility is given by $u(w, q) = \frac{1}{4} q^{\frac{1}{2}} w^2$, $v(w, r) = \kappa r$, with $\kappa \in \mathbb{R}_+$, $c(e) = \frac{1}{2}e^2$, and the quality of a social group $\mathscr{F}_k$ is determined by mean engagement (subject to $\underline{q} = 0$ and some arbitrarily large $\overline{q} > 0$). Let $I = \{[\underline{w}, \overline{w}]\}$. Welfare is evaluated according to U_0, i.e., $\lambda = 0$.

For $\overline{w} = 2$ and $\alpha = 0$, the (unique) equilibrium group quality is $q \approx 0.34$. For $\kappa = 10$, C2 is satisfied (Fig.1 **(a)**). Welfare is increasing in status concern for low α, but decreasing for larger values (Fig.1 **(b)**). For instance, at $\alpha' = 0.05$ ($q' \approx 0.72$), welfare is strictly greater than at $\alpha = 0$. While for $\alpha'' = 0.1$ ($q'' \approx 1.03$), despite the higher quality, welfare is lower. For low but positive α, status concern remedies inefficiently low engagement. While quality is increasing in α over this parameter range, from the perspective of the planner, who attaches no weight ($\lambda = 0$) to status, the increase in quality for large α is not sufficient to balance the increase in cost. This then causes a welfare loss. At lower κ, for instance at $\kappa = 2$ ($q \approx 0.34$ at $\alpha = 0$), welfare is increasing in α over the entire range of $\alpha \in [0, 0.1]$.

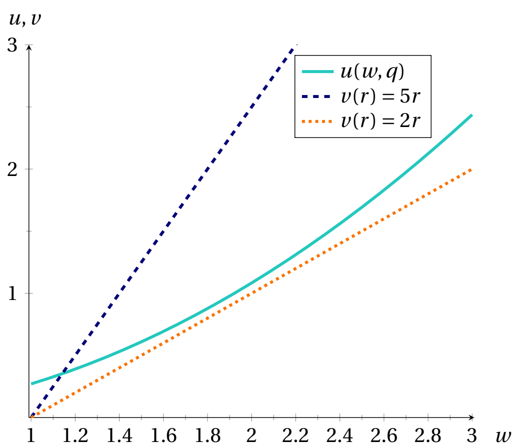
Qualitatively similar results obtain for $\overline{w} = 3$ and $\kappa = 10$. Due to the higher types, however, quality has a stronger effect on welfare, which is increasing over the entire range of $\alpha \in [0, 0.1]$. In contrast, for lower κ, status concern does not provide a sufficient incentive to increase engagement because of the higher equilibrium group quality at $\alpha = 0$, and hence greater $u(w, q)$. For example, C2 fails to hold at $\kappa = 2$ ($q \approx 1.17$) and welfare is decreasing in α for all $\alpha \in [0, 1]$.



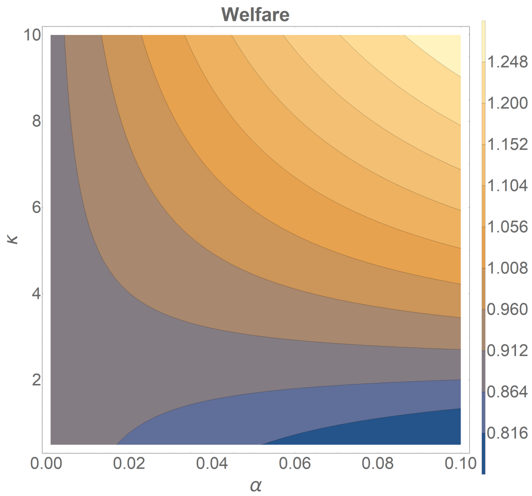
(a) $u(w,q), v(r)$, with $\overline{w} = 2$ and $q = \frac{49}{144} \approx 0.34$



(b) Welfare, $\overline{w} = 2$



(c) $u(w,q), v(r)$, with $\overline{w} = 3$ and $q = \frac{169}{144} \approx 1.17$



(d) Welfare, $\overline{w} = 3$

Figure 1: Panels **(a)** and **(c)** show u, v, for $\kappa \in \{2, 5\}$ and the respective equilibrium qualities, with $\overline{w} = 2$ **(a)** and $\overline{w} = 3$ **(c)**. Panels **(b)** and **(d)** show contour plots of welfare with $\lambda = 0$ for different α (horizontal axis), κ (vertical axis), and their corresponding equilibrium q. For $\overline{w} = 2$ and $\kappa = 2$, C2 holds. Welfare is increasing in α over the parameter range $\alpha \in [0, 0.1]$ (see **b**). For $\overline{w} = 3$ and $\kappa = 2$, $u(w,q) > v(r(w))$ for all $w \in [1,3]$, meaning C2 fails. Welfare is decreasing in α over the parameter range (**d**) and, in fact, for all $\alpha \in [0,1]$. For $\kappa = 5$, however, C2 holds and welfare is increasing in α.

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