Online Appendix

Supplementary Materials for "The Formation of Social Groups Under Status Concern"

Manuel Staab[1]

[1] *School of Economics, The University of Queensland, Brisbane, Australia, email: manuelstaab@gmail.com*

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1 Additional Results

1.1 Additional Results - Section 3

The following Proposition OA1 gives a clearer result for the effect of changes in α on sorting by restricting the analysis to cases where engagement does not affect quality. Recall that we say quality is independent of engagement, when quality is only determined by the (distribution over) types in a group, not their engagement choices (i.e., $q_k = \hat{q}_k$, if $F_k = \hat{F}_k$, independent of $\mathbf{e}$).

Proposition OA1. *If quality is independent of engagement, then for any U_α, with $\alpha \in (0,1]$, there exists a least upper-bound N_α on the number of social groups in equilibrium. N_α is (weakly) decreasing in α.*

Proof. As $\mathbb{N}$ has the least upper bound property, existence of an upper bound (Proposition 1) implies the existence of a least upper bound. It is shown next that this least upper bound is (weakly) decreasing in α. For a given $\alpha \in (0,1)$, denote the least upper bound by N_α. It was shown in the proof of Proposition 4 (main text) that if quality is independent of engagement and some group structure $(I, \mathbf{e}, \mathbf{p})$ can be provided in equilibrium for U_α, then for any $\hat{\alpha} < \alpha$, there exists an equilibrium $(I, \hat{\mathbf{e}}, \hat{\mathbf{p}})$.

Suppose now $N_\alpha > N_{\hat{\alpha}}$, for some $\hat{\alpha} < \alpha$. Then there must exist an equilibrium $(I, \mathbf{e}, \mathbf{p})$ for preferences U_α, such that $N_\alpha = |I| > N_{\hat{\alpha}}$. But this contradicts there being an equilibrium $(I, \hat{\mathbf{e}}, \hat{\mathbf{p}})$ for preferences $U_{\hat{\alpha}}$. The least upper-bound N_α must be weakly decreasing in α. □

1.2 Additional Results - Section 4

To systematically examine how status concern affects the welfare and revenue effects of sorting, the following result focuses on group structures that can be ordered in terms of the coarseness of their partition. Suppose $(I, \mathbf{e}, \mathbf{p})$ and $(I', \mathbf{e}', \mathbf{p}')$ are both equilibria. If I' is finer than I, meaning it allows for finer sorting, then $(I', \mathbf{e}', \mathbf{p}')$ is called a *more segregated* group structure. A general comparison of preferences with and without status concern is not straight-forward, since different utility levels lead to different engagement choices and can thus change the benefit from sorting through $c(e)$. Proposition OA2 shows that, abstracting from such effects by equalising engagement, status concern makes segregation less beneficial - both to a planner and a monopolist. The Proposition assumes engagement is identical across individuals and groups. This is, for instance, the case if we replace the functional form of $c(e)$ (Assumption 5) with one that has a sufficiently sharp kink at some $e^* \in \mathbb{R}_+$.

Proposition OA2. *Given Assumptions 1-4, suppose for every social group $\mathscr{F}_k$, $e^*_k(w) = e^* > 0$, for all $w \in [\underline{w}, \overline{w}]$. Then providing a more segregated equilibrium group structure achieves higher welfare under status concern ($\alpha > 0$) only if it achieves higher welfare without status concern ($\alpha = 0$). It achieves higher profit under status concern only if it achieves higher profit without status concern.*

Proof. To simplify notation, we re-write $U_\alpha(w, k, \mathscr{F}_k) = e^* \left[(1-\alpha)u(w, q_k) + \alpha v(w, r_k(w))\right] - c(e^*) - p_k = \tilde{u}(w, q_k) + \tilde{v}(w, r_k(w)) - p_k$, where $\tilde{u}(w, q_k) \equiv (1-\alpha)\big(e^* u(w, q_k) - c(e^*)\big)$ and $\tilde{v}(w, r_k(w)) \equiv \alpha\big(e^* v(w, r_k(w)) - c(e^*)\big)$. Note that for U_q, $\tilde{v}(w, r) = 0$ and hence $U_q(w, k, \mathscr{F}_k) = \tilde{u}(w, q_k) - p_k$.

Welfare maximisation: The proof demonstrates the contrapositive: if splitting a group is not beneficial for preferences U_q, it cannot be beneficial for U. As I' is finer than I, there are at least two social groups $\mathscr{F}_l$ and $\mathscr{F}_h$ under I', with the union of their supports equal to (a subset of) the support of some $\mathscr{F}_k$ under I. Suppose, for now, the union of their support exactly equals that of $\mathscr{F}_k$. Let the corresponding qualities be q_k, q_l and q_h. WLOG, assume $q_l < q_h$, noting that they can't be equal in equilibrium. It follows from Lemma 1 that $\underline{w}_k = \underline{w}_l < \overline{w}_l = \underline{w}_h < \overline{w}_h = \overline{w}_k$. If I' achieves lower welfare than I without status concern (U_q), then:

$$\int_{\underline{w}_l}^{\overline{w}_l} \tilde{u}(w, q_l) dF(w) + \int_{\underline{w}_h}^{\overline{w}_h} \tilde{u}(w, q_h) dF(w) \leq \int_{\underline{w}_l}^{\overline{w}_h} \tilde{u}(w, q_k) dF(w).$$

For some $r_l(w)$ and $r_h(w)$, this implies:

$$\begin{aligned} &\int_{\underline{w}_l}^{\overline{w}_l} \tilde{u}\big(w, q_l\big) + \tilde{v}\big(w, r_l(w)\big) dF(w) + \int_{\underline{w}_h}^{\overline{w}_h} \tilde{u}\big(w, q_h\big) + \tilde{v}\big(w, r_h(w)\big) dF(w) \\ \leq &\int_{\underline{w}_l}^{\overline{w}_l} \tilde{u}\big(w, q_k\big) + \tilde{v}\big(w, r_l(w)\big) dF(w) + \int_{\underline{w}_h}^{\overline{w}_h} \tilde{u}\big(w, q_k\big) + \tilde{v}\big(w, r_h(w)\big) dF(w), \end{aligned} \tag{1}$$

which compares welfare under status concern in social groups $\mathscr{F}_l$ and $\mathscr{F}_h$ to $\mathscr{F}_k$, when rank in $\mathscr{F}_k$ is assigned according to r_l and r_h. Actual welfare in $\mathscr{F}_k$ (with rank r_k) equals:

$$\int_{\underline{w}_l}^{\overline{w}_l} \tilde{u}\big(w, q_k\big) + \tilde{v}\big(w, r_k(w)\big) dF(w) + \int_{\underline{w}_h}^{\overline{w}_h} \tilde{u}\big(w, q_k\big) + \tilde{v}\big(w, r_k(w)\big) dF(w). \tag{2}$$

The final step is to show that (2) is larger than the right-hand side of (1). This is done using an intermediate step, demonstrating that any re-allocation of rank across types, as described by the change from $r_k(w)$ to $r_l(w)$ and $r_h(w)$, leads to a welfare loss.

Suppose there is only one type w^* but the measure of agents is the same as in the interval $[\underline{w}_l, \overline{w}_h]$, namely $\kappa \equiv F(\overline{w}_h) - F(\underline{w}_l)$. Suppose further that, despite the mass point, all ranks (from 0 to 1) are still allocated. Making use of the probability integral transform, we can establish that any distribution of ranks in a group is uniform. The welfare from such a group with (arbitrary) quality q can be written as:

$$\kappa \int_0^1 \tilde{u}(w^*, q) + \tilde{v}(w^*, r) dr.$$

The integral can be re-written in two alternative ways (ignoring κ for now). Firstly, for some $x \in (0, 1)$,

$$\int_0^1 \tilde{u}(w^*, q) + \tilde{v}(w^*, r) dr = \int_0^x \tilde{u}(w^*, q) + \tilde{v}(w^*, r) dr + \int_x^1 \tilde{u}(w^*, q) + \tilde{v}(w^*, r) dr.$$

And secondly, for the same x,

$$\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr = x\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr+(1-x)\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr.$$

Set $x = r_k(w^*)$. Multiplying by κ, we can obtain

$$\kappa\int_x^1 \tilde{u}(w^*,q)+v(w^*,r)dr = \int_{w^*}^{\overline{w}_h} \tilde{u}(w^*,q)+\tilde{v}(w^*,r_k(w))dF(w)$$
$$\kappa\cdot(1-x)\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr = \int_{w^*}^{\overline{w}_h} \tilde{u}(w^*,q)+\tilde{v}(w^*,r_h(w))dF(w),$$

as well as,

$$\kappa\int_0^x \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr = \int_{\underline{w}_l}^{w^*} \tilde{u}(w^*,q)+\tilde{v}(w^*,r_k(w))dF(w)$$
$$\kappa\cdot x\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr = \int_{\underline{w}_l}^{w^*} \tilde{u}(w^*,q)+\tilde{v}(w^*,r_l(w))dF(w).$$

The difference between this hypothetical and the actual welfare for types $w > w^*$ is:

$$\Delta_+^k \equiv \int_{w^*}^{\overline{w}_h} \tilde{u}\big(w,q\big)+\tilde{v}\big(w,r_k(w)\big)-\Big(\tilde{u}\big(w^*,q\big)+\tilde{v}\big(w^*,r_k(w)\big)\Big)dF(w),$$

$$\Delta_+^h \equiv \int_{w^*}^{\overline{w}_h} \big(\tilde{u}(w,q)+\tilde{v}(w,r_h(w))-\big(\tilde{u}(w^*,q)+\tilde{v}(w^*,r_h(w))\big)\big)\,dF(w).$$

From Assumption 1 (complementarity) and $r_k(w) \geq r_h(w)$, we can conclude that $\Delta_+^k \geq \Delta_+^h$, with the inequality strict for strict complementarity between type and status. An equivalent construction can be made for types $w < w^*$. The difference between the hypothetical and actual utilities can be expressed as:

$$\Delta_-^k \equiv \int_{\underline{w}_l}^{w^*} \big(\tilde{u}(w^*,q)+\tilde{v}(w^*,r_k(w))-\big(\tilde{u}(w,q)+\tilde{v}(w,r_k(w))\big)\big)\,dF(w),$$

$$\Delta_-^l \equiv \int_{\underline{w}_l}^{w^*} \big(\tilde{u}(w^*,q)+\tilde{v}(w^*,r_l(w))-\big(\tilde{u}(w,q)+\tilde{v}(w,r_l(w))\big)\big)\,dF(w).$$

Using the same argument as before, we can conclude that $\Delta_-^l \geq \Delta_-^k$. It follows that

$$\int_{\underline{w}_l}^{\overline{w}_h} \tilde{u}(w,q)+\tilde{v}(w,r_k(w))dF(w) = \kappa\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr+\Delta_+^k-\Delta_-^k$$
$$\geq \kappa\int_0^1 \tilde{u}(w^*,q)+\tilde{v}(w^*,r)dr+\Delta_+^h-\Delta_-^l$$
$$= \int_{\underline{w}_l}^{\overline{w}_l} \tilde{u}(w,q)+\tilde{v}(w,r_l(w))dF(w)$$
$$+\int_{\underline{w}_h}^{\overline{w}_h} \tilde{u}(w,q)+\tilde{v}(w,r_h(w))dF(w).$$

Again, the inequality is strict for strict complementarity between type and status. The reassignment of ranks caused by 'splitting' F_k lowers welfare. As the effect of a change in quality is the same for U_α and U_q, we can conclude that this split has a less positive effect on welfare for $\alpha > 0$. As any refinement can be written as an iteration of such binary splits, the result follows.

Profit maximisation: Take any equilibrium group structure $(I, \mathbf{e}^*, \mathbf{p})$ and a corresponding social group $\mathscr{F}_k$. It follows from Lemma 2 that $p_k = \tilde{u}(\underline{w}_k, q_k) + \tilde{v}(\underline{w}_k, 0) - \gamma$. The parameter γ is either the stand-alone utility $\underline{u}$ or, if there is a social group $\mathscr{F}_{k-1}$ 'below', $\gamma = \tilde{u}(\underline{w}_k, q_{k-1}) + \tilde{v}(\underline{w}_k, 1) - p_{k-1}$. Since any split of F_k (the support over types of $\mathscr{F}_k$) does not affect utility and prices in groups below, we can treat γ as a constant.

Suppose the monopolist instead offers $(I', \mathbf{e}^{*'}, \mathbf{p}')$, where I' is a refinement of I, with the support of F_k split into two, and identical otherwise. Let $\mathscr{F}_l$ and $\mathscr{F}_h$ be the resulting social groups (keeping e^* constant), where $\underline{w}_l = \underline{w}_k$, $\overline{w}_l = \underline{w}_h$, and $\overline{w}_k = \overline{w}_h$. It follows from IC that $p'_g = p_g$ for all groups with $\overline{w}_g \leq \underline{w}_k$. All other membership prices need to be adjusted, which determines the change in revenue. Notice that as $\mathscr{F}_k$ is split into $\mathscr{F}_l$ and $\mathscr{F}_h$, $\mathbf{p}'$ holds one additional price, which is taken to be p'_h. We can decompose the price changes into the effect caused by $p'_l - p_k$, which affects all prices $p_i \geq p_k$ in $\mathbf{p}$, and the effect of introducing a new price p'_h, which increases all prices $p_i > p_k$ by $p'_h - p'_l$.

IC requires $q_l < q_h$ and $p'_l < p'_h$. Because of separability, we can conclude that $p'_l - p_k = \tilde{u}(\underline{w}_k, q_l) - \tilde{u}(\underline{w}_k, q_k)$, which is the same for preferences U_α and U_q, as any split of F_k affects the price at $\underline{w}_k$ only through changes in quality, noting that $r_l(\underline{w}_l) = r_k(\underline{w}_l) = 0$. All prices $p_i > p_k$, if any, have to adjust by the same difference. Furthermore, due to the additional price p'_h, prices in the initial provision with $p_i > p_k$ need to further adjust by $p'_h - p'_l$. It follows from IC that $p'_h = p'_l + \tilde{u}(\underline{w}_h, q_h) - \tilde{u}(\underline{w}_h, q_l) - \left(\tilde{v}(\underline{w}_h, 1) - \tilde{v}(\underline{w}_h, 0)\right)$. The difference to prices $\mathbf{p}$ equals:

$$p'_h - p_k = p'_l - p_k + \tilde{u}(\underline{w}_h, q_h) - \tilde{u}(\underline{w}_h, q_l) - \left(\tilde{v}(\underline{w}_h, 1) - \tilde{v}(\underline{w}_h, 0)\right). \quad (3)$$

As shown before, $p_l' - p_k$ is the same with and without status concern, while $\tilde{v}(\underline{w}_h, 1) - \tilde{v}(\underline{w}_h, 0)$ is strictly positive under status concern (and 0 without). It follows then from (3), that the change in revenue from splitting any group is strictly larger without status concern. As any refinement can be written as a sequence of such binary splits, the result follows. □

1.3 Additional Results - Section 5

Proposition 6 (main text) provides a condition that ensures a given interval partition with $|I| = 2$ can be provided in equilibrium. C2 provides a substantially stronger condition that extends this to I with an arbitrary number of groups. However, as Example 3.2 (main text) demonstrates, it might not be satisfied by either the unconstrained welfare or profit maximising partition.

Condition C2. *An interval partition I satisfies C2 if there exist $e_k(w) = e_k^*(w)$, for all $k \in A \setminus \{\emptyset\}$ and $w \in [\underline{w}_1, \overline{w}]$, and corresponding $\{\mathscr{F}_k\}$, such that for any two $\mathscr{F}_l, \mathscr{F}_h \in \{\mathscr{F}_k\}$ with $\underline{w}_l \leq \overline{w}_h$, and all $w \in [\underline{w}, \overline{w}]$:*

$$0 \leq \frac{\partial}{\partial w}\Big[\underline{e}_h(w)\big((1-\alpha)u(w,q_h) + \alpha v(w,0)\big) - c(\underline{e}_h(w)) \\ - \Big(\overline{e}_l(w)\big((1-\alpha)u(w,q_l) + \alpha v(w,1)\big) - c(\overline{e}_l(w))\Big)\Big],$$

where $\overline{e}_l(w)$ is the optimal engagement of type w, for q_l and $r = 1$, and $\underline{e}_h(w)$ for q_h and $r = 0$.

If C2 holds, then the complementarity in w and q outweighs any (possible) negative effects from the interaction between w and r. It extends Assumption 2 to cases where a higher quality group delivers lower utility for some members. C2 ensures transfers needed for IC decline with type and can thus be achieved in a budget-balanced way. Lemma OA1 shows that C2 is indeed stronger than requiring $\hat{U}_\alpha(w, l, \mathscr{F}_l) - \hat{U}_\alpha(w, h, \mathscr{F}_h) \leq \hat{U}_\alpha(\underline{w}_h, l, \mathscr{F}_l) - \underline{u}$, for any two adjacent groups with $\overline{w}_l = \underline{w}_h$. Proposition OA3 provides the result that any I satisfying C2 can be provided.

Lemma OA1. *If an interval partition I satisfies C2, then for any two social groups $\mathscr{F}_h$ and $\mathscr{F}_l$, with $\overline{w}_l = \underline{w}_h$, we have $\hat{U}_\alpha(w, l, \mathscr{F}_l) - \hat{U}_\alpha(w, h, \mathscr{F}_h) \leq \hat{U}_\alpha(\underline{w}_h, l, \mathscr{F}_l) - \underline{u}$, for all $w \geq \underline{w}_h$.*

Proof. Let $\underline{\hat{U}}(w, h, \mathscr{F}_h) = \underline{e}_h^*(w)\big((1-\alpha)u(w,q_h) + \alpha v(w,0)\big) - c(\underline{e}_h^*(w))$, where $\underline{e}_h^*(w)$ is the optimal engagement for type w given q_h and $r_h(w) = 0$, i.e., it fixes the rank at 0. Observe that $\underline{\hat{U}}(w, h, \mathscr{F}_h) \leq \hat{U}(w, h, \mathscr{F}_h)$. It follows directly from C2 that for all $w \geq \underline{w}_h$,

$\hat{U}(w,l,\mathscr{F}_l) - \underline{\hat{U}}(w,h,\mathscr{F}_h) \leq \hat{U}(\underline{w}_h,l,\mathscr{F}_l) - \hat{U}(\underline{w}_h,h,\mathscr{F}_h) \leq \hat{U}(\underline{w}_h,l,\mathscr{F}_l) - \underline{u}$, and hence $\hat{U}(w,l,\mathscr{F}_l) - \hat{U}(w,h,\mathscr{F}_h) \leq \hat{U}(\underline{w}_h,l,\mathscr{F}_l) - \underline{u}$, as required. □

Proposition OA3. *For any interval partition I of some $[w_1,\overline{w}] \subseteq [\underline{w},\overline{w}]$, if I satisfies C2, then there exist $\mathbf{p}^r$ and $\mathbf{e}$, such that $(I,\mathbf{e},\mathbf{p}^r)$ is an equilibrium group provision.*

Proof. Let preferences be $\tilde{U}_\alpha(w,k,\mathscr{F}_k)$. We use the simplified notation: $U = \tilde{U}_\alpha$, where $u(w,q) \equiv (1-\alpha)\tilde{u}(w,q)$ and $v(w,r) \equiv \alpha\tilde{v}(w,r)$. By Assumption 1, C2 can only hold if there exists $e^*_k(w)$, for all $k \in A$, such that $\underline{w}_h \geq \overline{w}_l \implies q_h > q_l$. WLOG, assume that social groups are ordered such that $q_1 < q_2 < ... < q_n$. It follows from Lemma OA1 and Proposition 6 that the statement is true for $|I| \leq 2$. Suppose $|I| > 2$. Let $p_1(r)$ and $p_2(r)$ be defined as in the proof of Proposition 6. Define the price $p_k(r)$ of group $\mathscr{F}_k$, with $k > 2$, as follows:

$$\begin{aligned}
p_k(r_k(w)) =& e_k(\underline{w}_k)\left[u(\underline{w}_k,q_k) + v(\underline{w}_k,0)\right] - c(e_k(\underline{w}_k) - \underline{u} \\
&- \int_{\underline{w}_2}^{\underline{w}_k} \frac{\partial}{\partial w}\Big(e_{g(w)-1}(w)\left[u(w,q_{g(w)-1}) + v(w,1)\right] - c(e_{g(w)-1}(w))\Big)dw \\
&+ \int_{\underline{w}_k}^{w} \frac{\partial}{\partial w}\Big(\hat{U}(w,k,\mathscr{F}_k) - \hat{U}(w,k-1,\mathscr{F}_{k-1})\Big)dw,
\end{aligned}$$

where $w \in [\underline{w}_k,\overline{w}_k]$, and $e_k(w) = e^*_k(w)$. It is easily verified that for all $\mathscr{F}_k$ with $k > 1$, $\hat{U}(\underline{w}_k,k,\mathscr{F}_k) - p_k(0) = \hat{U}(\underline{w}_k,k-1,\mathscr{F}_{k-1}) - p_{k-1}(1)$, meaning $\mathbf{p}^r$ ensures indifference at the cut-off. It remains to be shown that IC holds for all other types, and revenues are non-negative.

First, it is shown that all types weakly prefer their group to all lower quality groups ('downward IC'). Observe that the utility a type w, with $g(w) = k$, obtains from joining some $l \leq k$, equals:

$$\begin{aligned}
&e_l(w)\cdot\left[u(w,q_l) + v(w,1)\right] - c(e_l(w)) - p_l(1) \\
=& \int_{\underline{w}_2}^{w} \frac{\partial}{\partial w}\Big(e_{\gamma(w)}(w)\cdot\left[u(w,q_{\gamma(w)}) + v(w,1)\right] - c(e_{\gamma(w)}(w))\Big)dw,
\end{aligned}$$

where $\gamma(w) = \max\{g(w)-1, l\}$. As type and quality are complements, this is maximised for $l = k$. Downward IC is satisfied. Next, it is shown that all types weakly prefer their group over all higher quality groups ('upward IC'). Consider a type w, with $g(w) = k$, joining a group $h > k$. They obtain utility $\hat{U}(w,h,\mathscr{F}_h) - p_h(0)$, while type $\underline{w}_h$ joining the same group obtains $\hat{U}(\underline{w}_h,h,\mathscr{F}_h) - p_h(0)$. The difference in utility from membership in $\mathscr{F}_h$ between both types thus equals:

$$\int_{w}^{\underline{w}_h} \frac{\partial}{\partial x}\Big(e_h(x)\cdot\left[u(x,q_h) + v(x,0)\right] - c(e_h(x))\Big)dx. \tag{4}$$

By definition of $\mathbf{p}^r$, the net utility difference between both types, when following g, equals:

$$\int_{w}^{\underline{w}_h} \frac{\partial}{\partial w}\Big(e_{g(x)-1}(x)\cdot\big[u(x,q_{g(x)-1})+v(x,1)\big]-c(e_{g(x)-1}(x))\Big)dx. \tag{5}$$

By C2, (4) is weakly greater than (5). This implies that if any upward deviation is beneficial and hence $\hat{U}(w,h,\mathscr{F}_h)-p_h(0)>\hat{U}(w,k,\mathscr{F}_k)-p_k(r_k(w))$, then $\hat{U}(\underline{w}_h,h,\mathscr{F}_h)-p_h(0)>\hat{U}(\underline{w}_h,h-1,\mathscr{F}_{h-1})-p_{h-1}(1)$. But this contradicts indifference at the cut-off, which is ensured by construction of $\mathbf{p}^r$. Upward IC is also satisfied.

Finally, to show that the sum of membership payments is non-negative, note that $p_1(1)>p_1(0)\geq 0$. Furthermore,

$$\begin{aligned} p_k(r)-p_1(1)=&\int_{\underline{w}_2}^{\underline{w}_k}\frac{\partial}{\partial w}\Big(\underline{e}_{g(w)}(w)\big[u(w,q_{g(w)}+v(w,0)\big]-c(\underline{e}_{g(w)}(w))\\ &\qquad -\hat{U}(w,g(w)-1,\mathscr{F}_{g(w)-1})\Big)dw\\ &+\int_{\underline{w}_k}^{r_k^{-1}(r)}\frac{\partial}{\partial w}\Big(\hat{U}(w,k,\mathscr{F}_k)-\hat{U}(w,k-1,\mathscr{F}_{k-1})\Big)dw, \end{aligned} \tag{6}$$

where again $\underline{e}_{g(w)}(w)$ is the optimal engagement for w given $q_{g(w)}$ and $r=0$. By C2:

$$\frac{\partial}{\partial w}\Big(\underline{e}_{g(w)}(w)\big[u(w,q_{g(w)}+v(w,0)\big]-c(\underline{e}_{g(w)}(w))-\hat{U}(w,g(w)-1,\mathscr{F}_{g(w)-1})\Big)\geq 0,$$

which implies both terms of (6) are non-negative. Prices, and thus revenue, are non-negative. □

2 Inequality and Redistribution

The benefits from sorting depend on the distribution of relevant characteristics in the population. This allows for efficiency gains from redistribution, a topic that has received particular attention in the context of income sorting. In Benabou (2000), for instance, benefits from redistribution arise from imperfect credit markets, while the political demand for redistribution is mediated by inequality. Gallice and Grillo (2020) specifically explore the role of inequality in two different dimensions: income and social class. And Levy and Razin (2015) investigate whether redistribution schemes that prevent sorting can be supported by a large majority. They find that while sorting is preferred for high inequality, full redistribution is favoured by a majority in more equal societies. This section briefly examines this issue when individuals have status concern. It reinforces the argument that redistribution can increase welfare but identifies a new channel for such gains: with status concern, transfers matter for the implementability of a group structure, allowing for welfare effects directly from the impact on sorting. In close analogy to Levy and Razin (2015) (henceforth 'LR'), it is demonstrated how this can reduce the set of individuals in favour of full redistribution and give segregation large majority support (Example OA1). Positional concerns thus offer an additional explanation why transfers can be found even in segregated environments, why such transfers receive support from individuals that significantly benefit from segregation, and why transfers are not always limited or even aimed at the poorest members of society (Acemoglu et al., 2015; Brady and Bostic, 2015). Furthermore, redistribution that allows for sorting does not necessarily lead to more equal outcomes. Due to the simplicity of the model and the complexity of the issue, these implications should be treated with caution. Nevertheless, they might highlight some aspects that warrant further investigation.

In LR, agents can sort into groups and the quality of each group is determined by the average type. Preferences are multiplicative in type and quality, and additive in payments, thus satisfying Assumptions 1-4. The majority is said to prefer full redistribution, if the utility of the mean type is higher with full redistribution than any other incentive compatible sorting structure. LR identify a simple condition on the distribution that ensures full redistribution is preferred by a majority.[1] With analogous preferences that include status concern, it is demonstrated here that sorting might be preferred by such a majority even if a society is relatively equal and the condition is met. Given the quality function, lower inequality implies lower quality differences between groups. The benefit from sorting can then be less than the membership price that must be charged to achieve incentive compatibility. Status concern, however, further reduces the utility difference at the cut-off between two groups. This reduces the price of sorting from an individual's perspective since the membership price of the higher quality group needs to be lower. As this does not necessarily equally reduce the benefit from sorting, it can tilt the trade-off against full redistribution. However, rank-based prices and subsidies might be necessary for incentive compatibility. Such limited redistribution can lead a majority that extends beyond the mean to prefer sorting. While (some) redistribution might also affect the majority preference without status concern, transfers considered here are 'minimal' in the sense that they are strictly necessary to sustain sorting. For instance, in Example OA1, types around the mean do not receive any subsidies, meaning they are not being 'bribed'. As transfers are necessary to maintain the segregated structure, even individuals paying for them might be in favour. Without status concern, individuals paying towards subsidies would always prefer a group structure without them.

Definition 1 formalises the notion of (majority) preference for group structures. To

[1] See 'Condition 1' in Levy and Razin (2015), which requires $\frac{w}{E[w]} \geq F(w),\ \forall w \in [\underline{w}, E[w])$.

ensure comparability with LR, the focus lies on the mean type ($E[w]$). This abstracts from the incentives below average types might have to favour group structures involving redistribution.

Definition 1. *A type w is said to prefer a group structure $(I, \mathbf{e}, \mathbf{p}^r)$ to $(\hat{I}, \mathbf{e}, \hat{\mathbf{p}}^r)$ if:*

$$U_\alpha(w, g(w), \mathscr{F}_{g(w)}) \geq U_\alpha(w, \hat{g}(w), \hat{\mathscr{F}}_{\hat{g}(w)}). \tag{7}$$

A majority is said to prefer $(I, \mathbf{e}, \mathbf{p}^r)$ to $(\hat{I}, \hat{\mathbf{e}}, \hat{\mathbf{p}}^r)$ if (7) holds for all $w \in [\underline{w}, E[w]]$ or $w \in [E[w], \overline{w}]$.

The following Example OA1 presents an instance when sorting increases aggregate welfare but requires transfers. To make it comparable to LR, it abstracts from engagement choice (i.e., $e_k(w) = 1$ for all w and k, and $c(1) = 0$). For simplicity, group structures are thus referenced by a pair $(I, \mathbf{p})$, dropping engagement. Sorting is compared to a group structure where all utility is equalised through prices and subsidies, which is referred to as *full redistribution*. Note that this necessarily requires an integrated group, as no sorting could be achieved in equilibrium with such transfers. If types relate to income, full redistribution could also imply a redistribution before sorting takes place; an equalization of types. The example is chosen such that these two interpretations are outcome equivalent (assuming that in a group of equal types, agents obtain rank 1/2).

Example OA1. Suppose types are distributed according to a (truncated) Pareto distribution with shape parameter $s = 1$ over $[1, 3]$. The mean type is $E[w] \approx 1.65$. Utility is given by $u(w, q) + v(w, r)$, with $u(w, q) = qw$ and $v(w, r) = (r - \frac{1}{2})w$ and $\underline{u} = 0$. Note that utility is rescaled to abstract from α, i.e., $u(w, q) + v(w, r) = \frac{1}{2}\tilde{u}(w, q) + \frac{1}{2}\tilde{v}(w, r)$, where $\tilde{u}(w, q) = 2u(w, q)$ and $\tilde{v}(w, r) = 2v(w, r)$. The quality of a social group $\mathscr{F}_k$ equals the mean type within that group, i.e., $q_k = \int w dF_k$.
Consider partitions $I = \{[1, 1.25], [1.25, 3]\}$ and $\hat{I} = \{[1, 3]\}$. The quality of the single group in the partition $\hat{I}$ equals $E[w]$. Let $\hat{u}$ bet the utility from $\hat{I}$, when all surplus is equally redistributed, meaning that

$$\hat{p}(r) = u(w^*(r), E[w]) + v(w^*(r), r) - \int_0^1 u(w^*(r), E[w]) + v(w^*(r), r) dr,$$

with $w^*(r)$ the type such that $F(w^*(r)) = r$. The group structure $(\hat{I}, \hat{\mathbf{p}}^r)$ thus represents full redistribution, equalising utility across all types to $\hat{u}$. As $\hat{u} > \underline{u}$, this is an equilibrium.
Denote group qualities of the segregated partition I by q_1 and q_2. Suppose, for now,

membership payments are 0, but prices $\mathbf{p}^r$ of $(I, \mathbf{p}^r)$ include the following subsidies:

$$s_2(r) = \max\Big[0, u(w_2^*(r), q_1) + v(w_1^*(r), 1) - \big(u(w_2^*(r), q_2) + v(w_2^*(r), r)\big)\Big]$$
$$s_1(r) = s_2(0) + u(w_1^*(r), q_2) + v(w_1^*(r), 0) - \big(u(w_1^*(r), q_1) + v(w_1^*(r), r)\big),$$

where again $w_k^*(r)$ denotes the type that achieves rank r in group $\mathscr{F}_k$. Figure 1 plots expected utility with prices defined as $p_k(r) = -s_k(r)$. As can be seen from the graph, these are the minimum subsidies that achieve incentive compatibility. It is easily verified that $g(E[w]) = 2$ and $s_2(r_2(E[w]) = 0$, meaning that the mean type is a member of group $\mathscr{F}_2$ and does not receive subsidies. The aggregate net benefit from joining group $\mathscr{F}_2$ instead of $\mathscr{F}_1$ of all types above the mean equals $\int_{E[w]}^{\overline{w}} u(x, q_2) + v(x, r_2(x)) dF(x) \approx 0.45$. The total subsidies amount to $\int_0^1 s_1(r) + s_2(r) dr \approx 0.20$. We can thus find incentive compatible membership prices $p_2(r)$ for types $w > E[w]$ that achieve budget balance. Figure 2 shows such a pricing schedule and Figure 3 confirms IC. ⋄

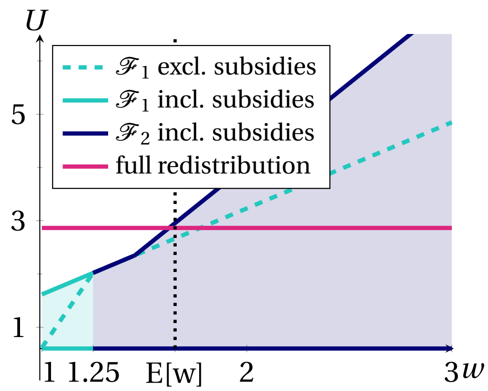

Figure 1: Utility as a function of type for different group choices (Ex.OA1). Shaded areas indicate group membership for the segregated group structure $(I, \mathbf{p}^r)$. The utility of the mean type $(E[w])$ is higher under segregation than full redistribution.

Figure 2 demonstrates that with status concern, membership prices/subsidies are not necessarily monotone in type or group quality. To achieve sorting, subsidies may need to be directed towards intermediate types with low status. Types close to but above $\overline{w}_1 = 1.25$ (must) receive strictly larger subsidies than types just below $\overline{w}_1$, even though they are members of a higher quality group. Interestingly, not all net contributors would prefer to abolish redistribution. Incentive compatibility and budget balance can be achieved such that at least some individuals who pay for the subsidies prefer sorting to an integrated group structure without transfers (Figure 3). As can be easily verified, all segregated equilibrium group structures require transfers. While contributors would necessarily prefer to shift the burden to other types, there exists no equilibrium without transfers that is preferred by all contributors. Without status concern, all net contributors would strictly prefer a group structure that entails no zero-sum transfers.

As the mean type achieves higher utility under $(I, \mathbf{p}^r)$ than $(\hat{I}, \hat{\mathbf{p}}^r)$, the majority prefers sorting to full redistribution. The distribution of types satisfies Condition 1 of LR, which implies that without status concern, full redistribution would be preferred.

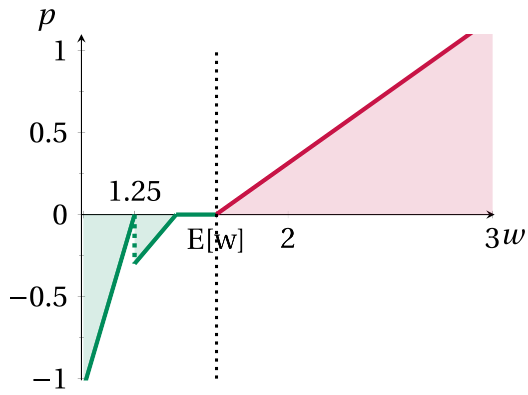

Figure 2: Transfers of the equilibrium group structure $(I, \mathbf{p}^r)$ that achieve budget balance. Positive values correspond to membership payments, negative ones to subsidies. The discontinuity equalises utility for the cut-off type between both groups.

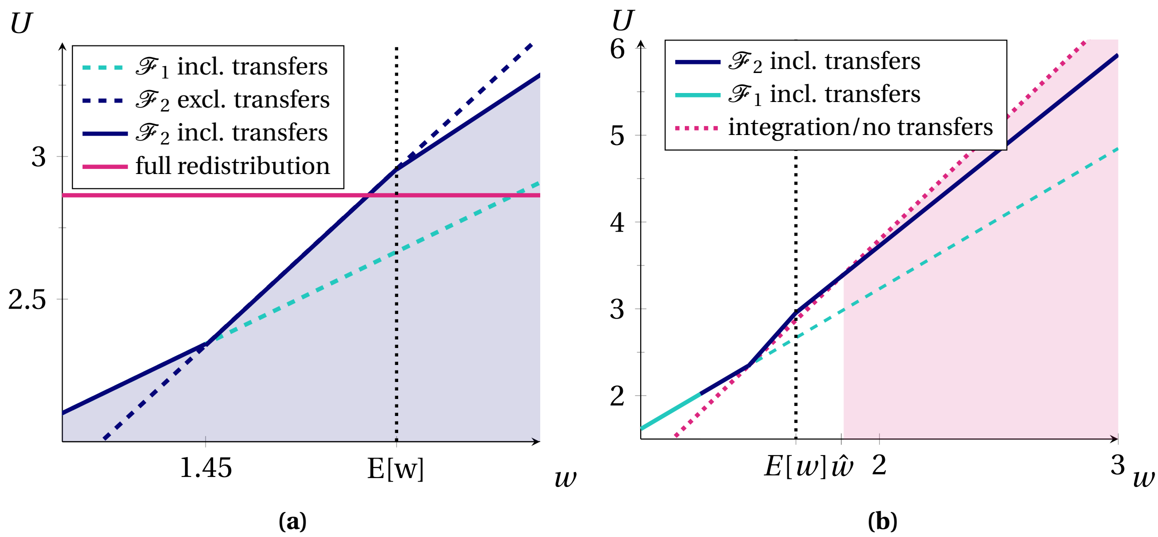

Figure 3: Utility as a function of type (Ex.OA1) with budget balanced, incentive compatible prices/subsidies. Panel (**a**) shows types up to $w = 1.45$ receive subsidies, while all types above $E[w]$ make payments. Utility of the mean type ($E[w]$) is higher under segregation than full redistribution. Panel (**b**) shows types below $\hat{w}$ prefer segregation over integration without transfers (non-shaded region). Types between $E[w]$ and $\hat{w}$ contribute to transfers and yet prefer segregation. While IC determines the subsidies, the payments to finance these are not uniquely determined. Other pricing schedules can achieve qualitatively equivalent outcomes.

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